Déjà Vu, It's Algebra 2!
Lesson 09
Matrix Multiplication

Last week we looked at multiplying a matrix by a *scalar*. Today we will look at multiplying two matrices together to obtain a *matrix product*.

☞ Two matrices A and B can be multiplied to get AB only if the number of columns of A equals the number of rows in B.

☞ The product of an $m \times n$ and an $n \times p$ matrix is an $m \times p$ matrix.

☞ An $m \times n$ matrix A can be identified using the notation $A_{m \times n}$

Example:

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 6 & -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 4 & 8 & 7 \\ 3 & 6 & 9 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix}$$

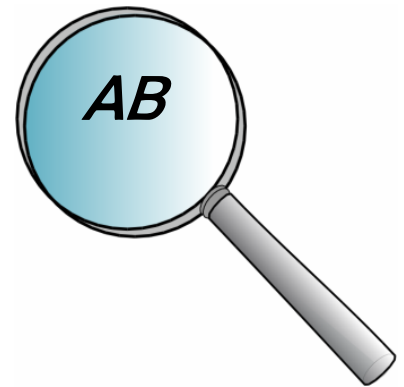$$\begin{array}{ccc} A & B & AB \\ 2 \times 3 & 3 \times 4 & = 2 \times 4 \end{array}$$

Example:

$$P = \begin{bmatrix} 1 & 5 \\ 8 & 8 \\ 9 & 0 \end{bmatrix} \qquad Q = \begin{bmatrix} 2 & 4 & -3 & 6 & 1 \\ 5 & 8 & 2 & 7 & 0 \\ -1 & 8 & 4 & 7 & 2 \end{bmatrix}$$

$$\begin{array}{ccc} P & Q & PQ \\ & & undefined \\ 3 \times 2 & 3 \times 5 & \\ & & 2 \neq 3 \end{array}$$

So now we know how to determine if the matrix product exists, but how do we actually FIND it?

We simply add the products of consecutive entries of the rows of the first matrix and the columns of the second matrix.

Example:

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

Find the following matrix products if possible.

$WX =$

$XW =$

$XY =$

$YX =$

Notice in the previous examples that $WX \neq XW$ and $XY \neq YX$. This means that

MATRIX MULTIPLICATION
IS
NOT COMMUTATIVE!!!

Even though the operation might be defined either way, they do not necessarily yield the same matrix, even if the resultant matrix is the same dimension.

Last time, we looked at the additive identity matrix, today, we will look at the multiplicative identity matrix.

The multiplicative identity matrix, I, is any square matrix with each entry along the main diagonal equal to 1 and all other entries equal to 0.

$$I_{2 \times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

For $A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find AI, IA and compare them.

$$AI = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$IA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix} =$$

So for any square matrix $A_{m \times m}$ and identity matrix $I_{m \times m}$,

$$A = AI = IA$$

Matrix multiplication is only commutative when the multiplicative identity matrix is one of the factors!

Déjà RE-Vu

Two stores held sales on their videos and DVDs, with prices shown below. Use the sales data to determine how much money each store brought in from the sale Saturday.

Sales Prices	Videos	DVDs
Video World	$8.95	$11.95
Star Movies	$7.50	$12.50

Total Sales	Friday	Saturday	Sunday
Videos	23	31	25
DVDs	40	48	42

References:

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