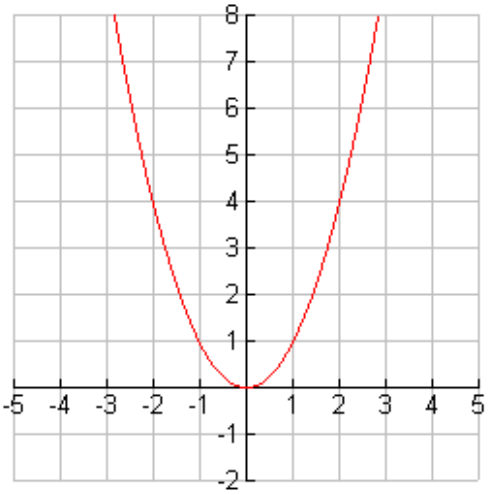
Déjà Vu, It's Algebra 2!

Lesson 12

Quadratic Functions: Reflections & Dilations, Roots, Max & Mins

REFLECTIONS

of $f(x) = x^2$

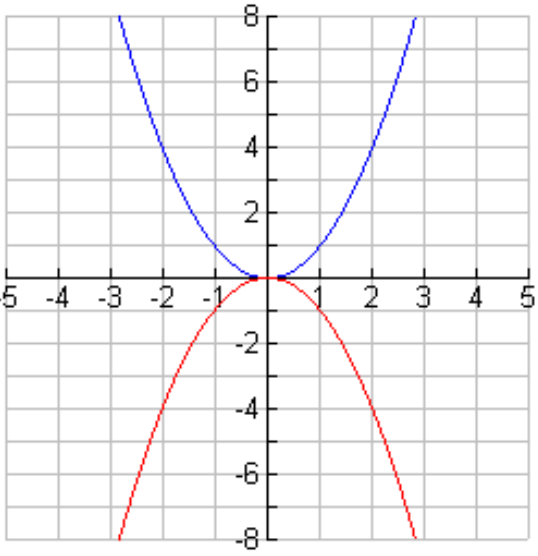


Reflection across *y*-axis

$f(-x) = (-x)^2$

Input values exchange

The vertex is a MINIMUM y-value of the graph



Reflection across *x*-axis

$-f(x) = -x^2$

Output values exchange

The vertex is a MAXIMUM y-value of the graph

DILATIONS: STRETCHES & COMPRESSIONS

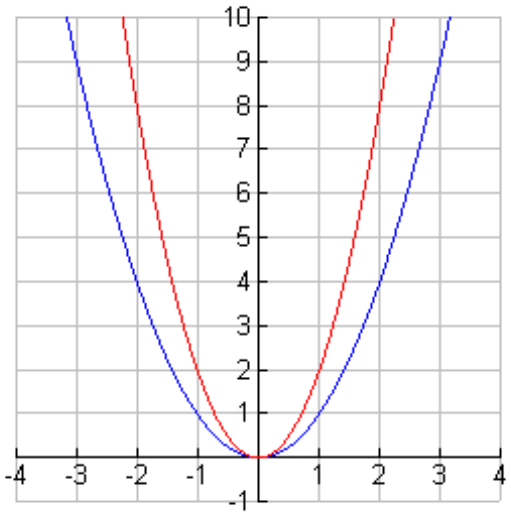
of $f(x) = x^2$

Vertical Dilation for $a > 0$

$f(x) = ax^2$

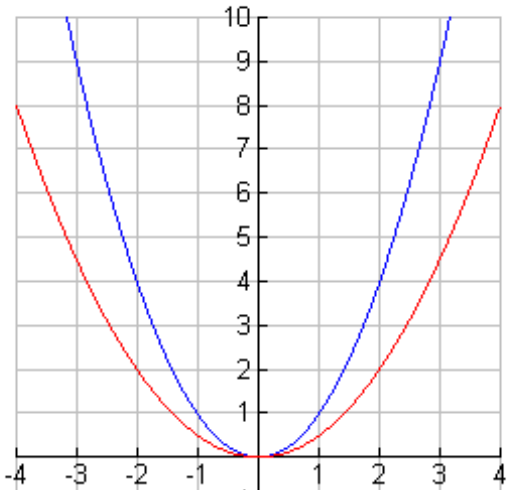
If $a > 1$
Vertical Stretch
away from x-axis

Ex) $g(x) = 2x^2$



If $0 < a < 1$
Vertical Compression
towards x-axis

Ex) $g(x) = \frac{1}{2}x^2$

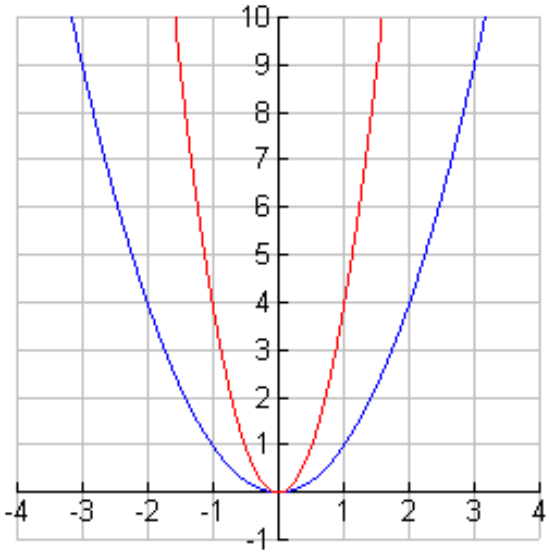


Horizontal Dilation for $b > 0$

$f(bx) = (bx)^2$

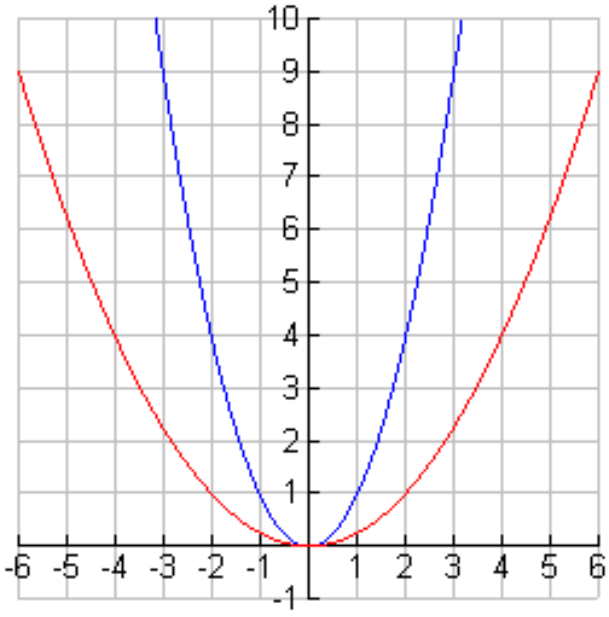
If $b > 1$
Horizontal compression towards y-axis

Ex) $g(x) = (2x)^2$



If $0 < b < 1$
Horizontal stretch away from y-axis

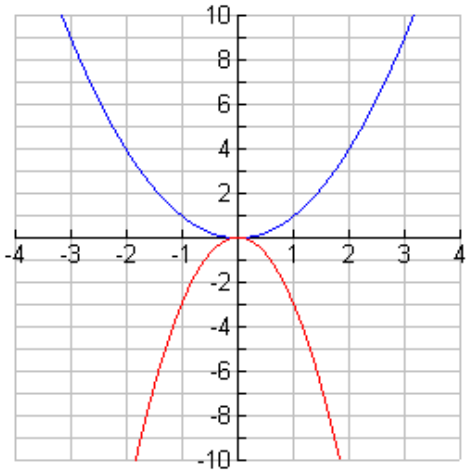
Ex) $g(x) = \left(\frac{1}{2}x\right)^2$



Example:

Sketch the graph of $f(x) = -3x^2$

Vertical stretch by a factor of 3 and an x-axis reflection in either order.



Mathematical "Synonyms"

- x-intercpets of the graph of a parabola
- roots of a quadratic function, $f(x)$
- zeros of a quadratic, $f(x)$
- solutions to the equation $f(x) = 0$

These values are generally more difficult to find than the y-intercpets, are much more meaningful, in terms of real-world applications.

Factoring is one way to solve quadratic equations:

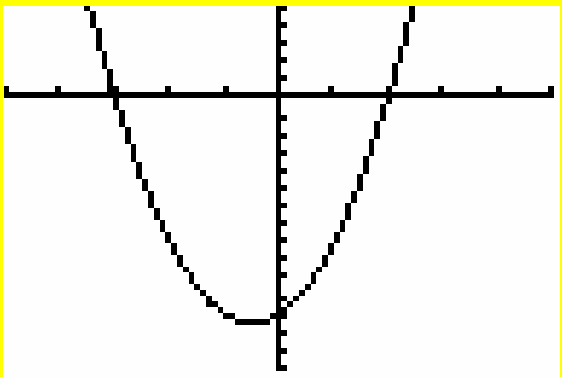
Example:

Find the zeros of $f(x) = 2x^2 + 2x - 12$

$$f(x) = 2\left(x^2 + x - 6\right) = 0$$

$$2(x+3)(x-2) = 0$$

$$x = -3, 2$$



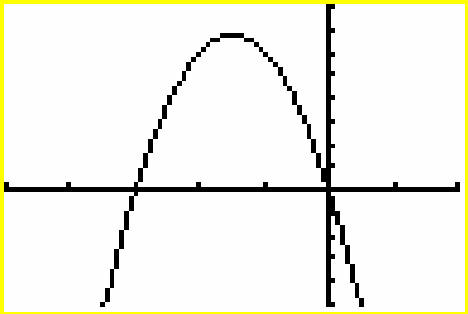
Example:

Find the roots of the following equation.

$g(x) = -3x^2 - 9x$

$g(x) = -3x(x+3) = 0$

$x = 0, -3$



Example:

Find the x-intercepts of the following equation.

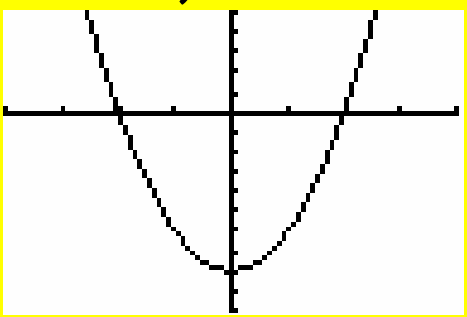
$h(x) = 2x^2 - 8$

$h(x) = 2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = -2, 2$



Déjà RE-Vu

Application

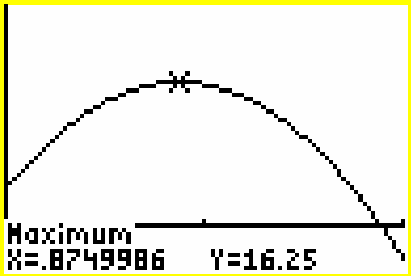
The height and velocity of a ball thrown straight up with an initial velocity of 28 feet per second from an initial height of 4 feet can be modeled by the following respective equations:

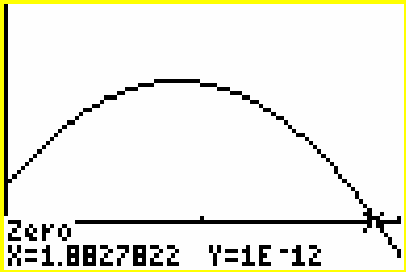
$$h(t) = -16t^2 + 28t + 4$$

$$v(t) = -32t + 28$$

a) What is the maximum height of the ball?
b) At what time does the ball reach its maximum height?
c) How long is the ball in the air?
d) What is the velocity of the ball as it hits the ground?

Answers:





a) Max height of 16.25 feet
b) Max height at 28/32 = 0.875 seconds
c) Ball hit the ground about 1.883 seconds after it is thrown
d) The velocity of the ball as it hits the ground is v(1.883) = -32.249 feet per second

References:
All images created with TI-Interactive software or TI-83+ calculator

For more information on applications of parabolas, check out the following website:
http://www.pen.k12.va.us/Div/Winchester/jhhs/math/lessons/calc2004/apppara b.html