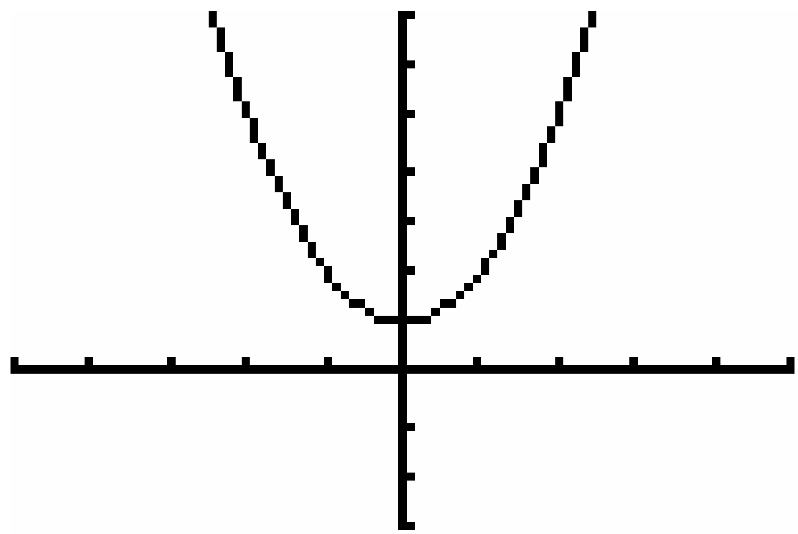
# *Déjà Vu, It's Algebra 2!*

# Lesson 13

# Complex numbers and Imaginary roots

## The Fundamental Theorem of Algebra:

*If a quadratic equation does not have any real x-intercepts, then it has all imaginary roots.*



**Example:**

$f(x) = x^2 + 1$

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

We define the square root of negative one to be the imaginary unit, *i*.

$$\sqrt{-1} = i$$

Electrical engineers use the imaginary unit (which they represent as j ) in the study of electricity to avoid confusion with electric current, traditionally denoted by "I". The symbol used to denote electric current is the capital letter "I", from the German word "Intensität", which means "intensity".

In mathematics, the term "imaginary" was first used by Rene Descartes as a derogatory term the symbol "i" was first introduced by Euler in 1748, possibly because "i" is the first letter of the Latin word "Imaginarius". (No prizes for guessing what that means!)

Despite being called "imaginary", imaginary numbers are just as "real" as the so-called "real numbers":

* Fractions such as 2/3 and 1/4 are meaningless to somebody counting, say, the number of number of people in a room; yet, they make sense to someone who is trying to cut a pie into pieces in a given ratio of sizes.

* Negative numbers like -453.6 and -21 don't make sense while talking about the age of a person, but have a large number of applications when dealing with money, in contexts such as debt, the change in the value of a share, etc.

Similar is the case with imaginary numbers- for most human tasks, they have no meaning, but they have concrete applications in various sciences and related fields, such as signal-processing, electromagnetism, quantum mechanics and cartography.

I'll illustrate this with an example: in electrical engineering, the values of electric current and voltage are sometimes expressed as imaginary numbers or complex numbers with non-zero imaginary parts, called "phasors". Even though these currents are supposedly "imaginary" (from the mathematical perspective), they can cause _real_ harm to people or damage to equipment!

# Example:

## Find the zeros of $g(x) = 9x^2 + 25$

$$9x^2 + 25 = 0$$

$$9x^2 = -25$$

$$x^2 = -\frac{25}{9}$$

$$x = \pm\sqrt{-\frac{25}{9}}$$

$$x = \pm\frac{5}{3}i$$

# Simplify:

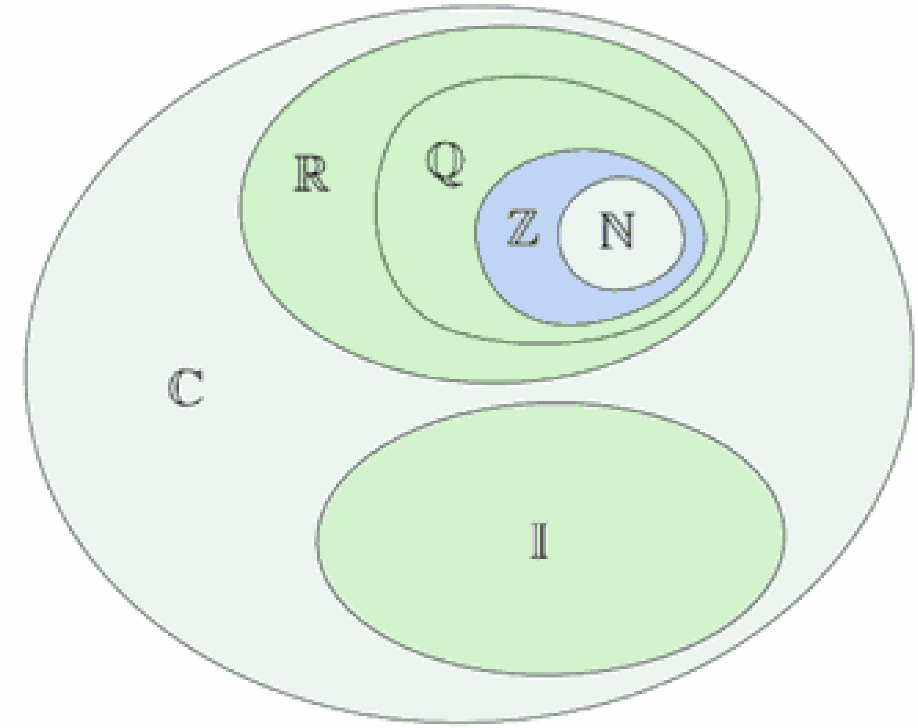
$-\sqrt{75}$

$\dfrac{-\sqrt{42}}{\sqrt{-3}}$

$5\sqrt{-32}$

# Complex Numbers:

The Set of Complex numbers are the largest set of numbers used in mathematics are composed of all combinations of real and imaginary numbers. We use the symbol $\mathbb{C}$ to denote the set.




More precisely, a complex number is one that can be written in the form $a + bi$, where $i$ is the imaginary unit and $a, b \in \mathbb{R}$

$a$ is called the REAL PART
$b$ is called the IMAGINGARY PART

## Examples:

$-4$ $\quad -5i \quad 2-3i \quad -i\sqrt{5}-7$

# Working with complex numbers: Powers of $i$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i = -\sqrt{-1}$

$i^4 = 1$

$i^5 = i = \sqrt{-1}$

$\vdots$

$i^0 = 1$

$i^{-1} = \frac{1}{i} = \frac{i^4}{i} = i^3$

$i^{-2} = \frac{1}{i^2} = \frac{i^4}{i^2} = i^2 = -1$

$\vdots$

Try this: $i^{263} =$

$263 \div 4 = 65$ remainder 3

$i^{263} = i^3 = -i$

263

We can also perform arithmetic with complex numbers. Let $u = 2+3i$, $v = -1+5i$, and $\bar{u} = 2-3i$ (the conjugate of $u$)

Simplify. Write each answer in standard complex form, $a+bi$

$u + v =$ 1+8i

$-3v - u =$ 1-18i

$uv =$ -17+7i

$u\bar{u} =$ 13

$\frac{v}{u} =$ 1+i

## Example:

Find the roots of the following quadratic function.

$$p(x) = x^2 + 4x + 10$$

By completing the square

$(x+2)^2 + 6 = 0$

$x + 2 = \sqrt{-6}$ or $x + 2 = -\sqrt{-6}$

$x = -2 + i\sqrt{6}$ or $x = -2 - i\sqrt{6}$

$x = -2 \pm i\sqrt{6}$

Or

By quadratic formula:

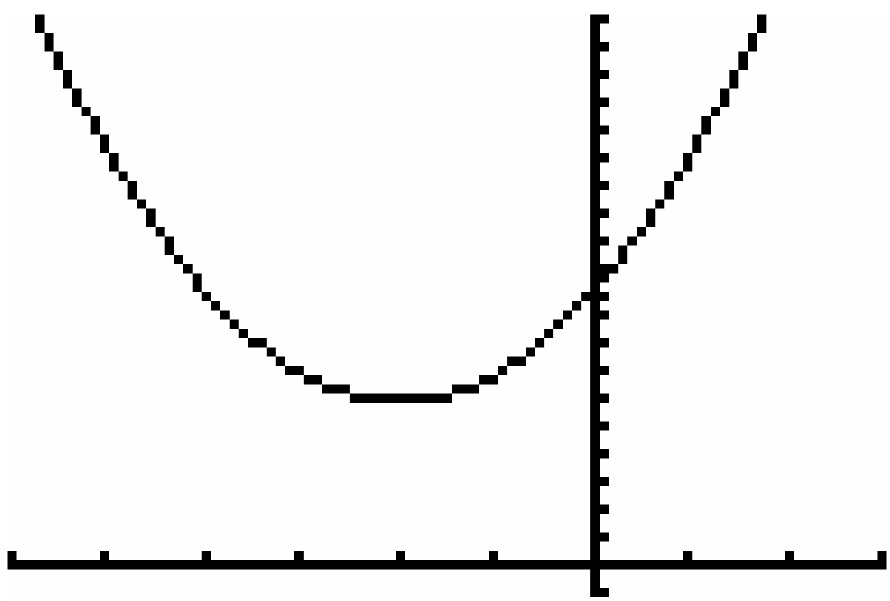
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(10)}}{2}$$

$$x = \frac{-4 \pm \sqrt{-24}}{2} = \frac{-4 \pm 2i\sqrt{6}}{2}$$

$$x = -2 \pm i\sqrt{6}$$

Notice the two imaginary roots occur in conjugate pairs!



# *Déjà RE-Vu*

## Solve the following quadratic equation using each method: factoring, completing the square, using the quadratic formula, and graphically.

$$2x^2 + 14x + 24 = 0$$

## Factoring:

$$2\left(x^2 + 7x + 12\right) = 0$$
$$2(x+4)(x+3) = 0$$
$$x = -4, -3$$

## Complete the Square:

$$2\left(x^2 + 7x + \left(\frac{7}{2}\right)^2\right) - 2\left(\frac{7}{2}\right)^2 + 24 = 0$$
$$2\left(x + \frac{7}{2}\right)^2 = -24 + \frac{49}{2}$$
$$\left(x + \frac{7}{2}\right)^2 = -12 + \frac{49}{4}$$
$$x + \frac{7}{2} = \pm\sqrt{\frac{1}{4}}$$
$$x = -\frac{7}{2} \pm \frac{1}{2}$$
$$x = -4, -3$$

# Quadratic Formula:

$$2x^2 + 14x + 24 = 0$$

$$a = 2,\ b = 14,\ c = 24$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

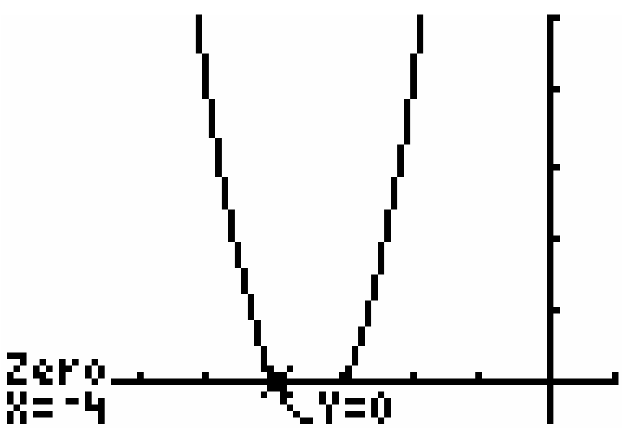
$$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(24)}}{2(2)}$$

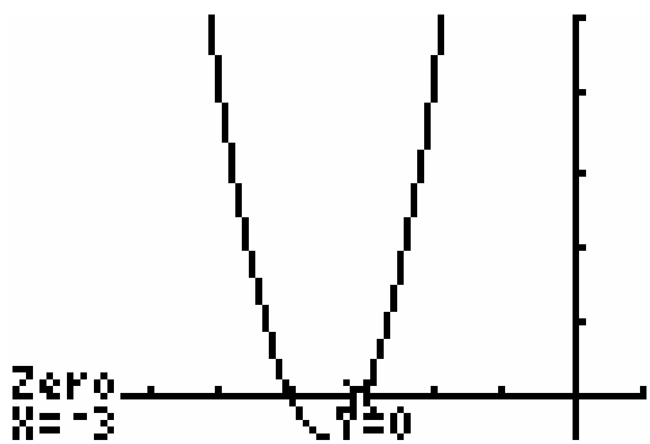
$$x = \frac{-14 \pm \sqrt{196 - 192}}{4}$$

$$x = \frac{-14 \pm 2}{4}$$

$$x = -4, -3$$

# Graphing:



# References:
# All images TI-83+ calculator

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