Déjà Vu, It's Algebra 2!

Lesson 13

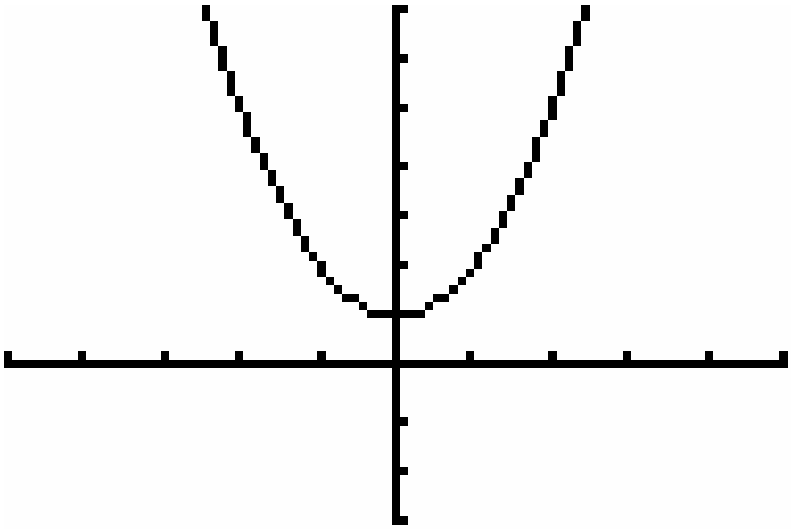
Complex numbers and Imaginary roots

The Fundamental Theorem of Algebra:

If a quadratic equation does not have any real x-intercepts, then it has all imaginary roots.

Example:

$f(x) = x^2 + 1$



We define the square root of negative one to be the imaginary unit, *i*.

$$\sqrt{-1} = i$$

Example:

Find the zeros of $g(x) = 9x^2 + 25$

Simplify:

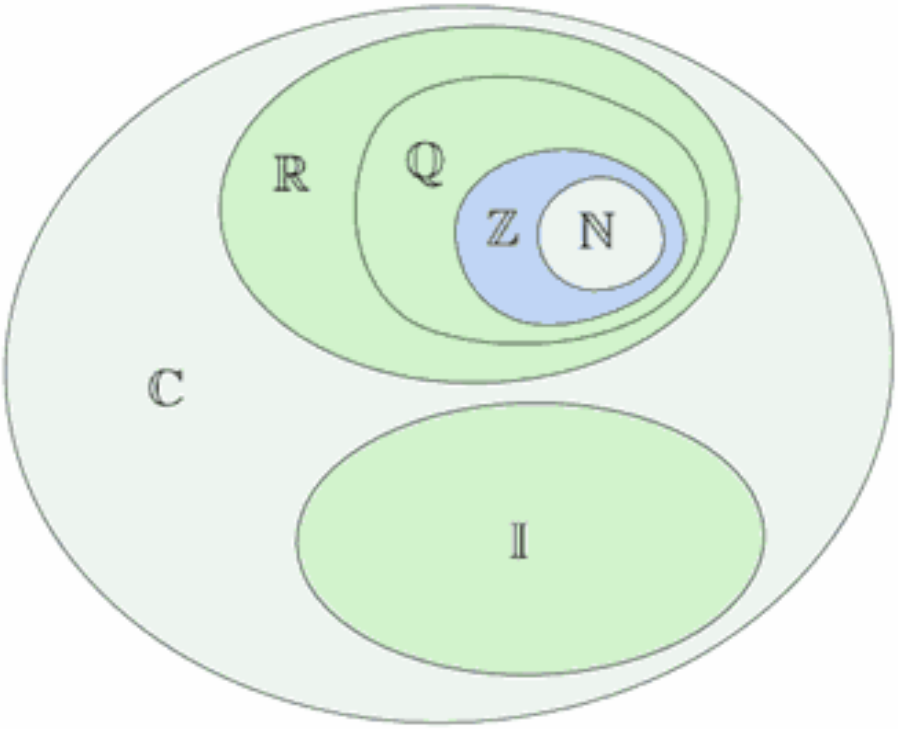
$-\sqrt{75}$

$\dfrac{-\sqrt{42}}{\sqrt{-3}}$

$5\sqrt{-32}$

Complex Numbers:

The Set of Complex numbers are the largest set of numbers used in mathematics are composed of all combinations of real and imaginary numbers. We use the symbol $\mathbb{C}$ to denote the set.



More precisely, a complex number is one that can be written in the form $a + bi$, where i is the imaginary unit and $a, b \in \mathbb{R}$

a is called the REAL PART
b is called the IMAGINARY PART

Examples:

-4 -5i 2-3i $-i\sqrt{5}-7$

Working with complex numbers: Powers of *i*

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i = -\sqrt{-1}$

$i^4 = 1$

$i^5 = i = \sqrt{-1}$

$\vdots$

$i^0 = 1$

$i^{-1} = \frac{1}{i} = \frac{i^4}{i} = i^3$

$i^{-2} = \frac{1}{i^2} = \frac{i^4}{i^2} = i^2 = -1$

$\vdots$

Try this: $i^{263} =$

We can also perform arithmetic with complex numbers. Let $u = 2 + 3i$, $v = -1 + 5i$, and $\overline{u} = 2 - 3i$ (the conjugate of u)

Simplify. Write each answer in standard complex form, $a + bi$

$u + v =$

$-3v - u =$

$uv =$

$u\overline{u} =$

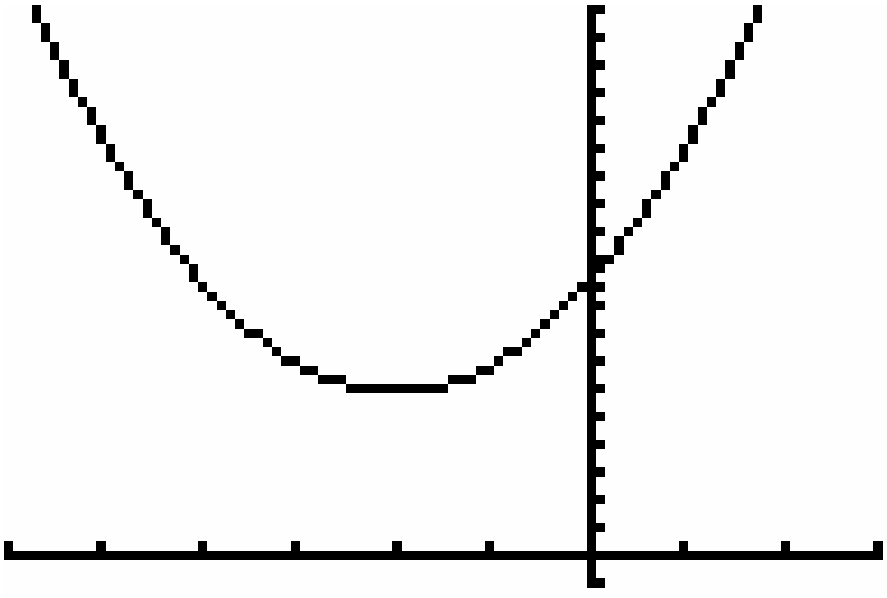
$\dfrac{v}{u} =$

Example:

Find the roots of the following quadratic function.

$p(x) = x^2 + 4x + 10$

Notice the two imaginary roots occur in conjugate pairs!



Déjà RE-Vu

Solve the following quadratic equation using each method: factoring, completing the square, using the quadratic formula, and graphically.

$$2x^2 + 14x + 24 = 0$$

Factoring:

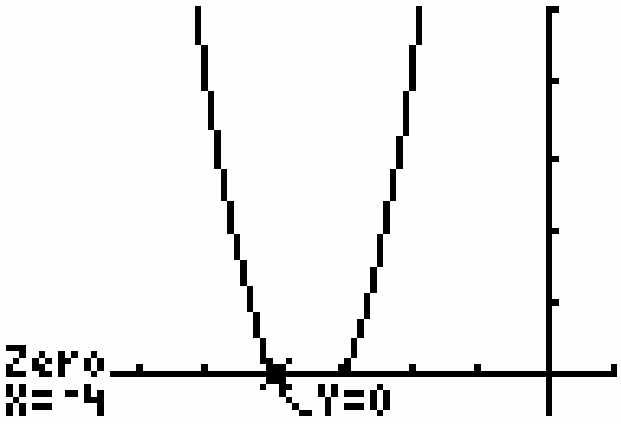
Complete the Square:

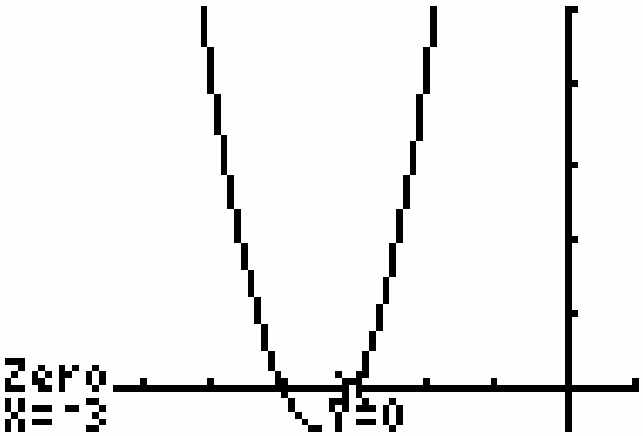
Quadratic Formula:

$$2x^2 + 14x + 24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphing:





References:
All images TI-83+ calculator

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