Déjà Vu, It's Algebra 2!

Lesson 14

Polynomials: Addition, Subtraction, & Multiplication

A polynomial is an expression that consists of adding or subtracting a combination of numbers and variables. The variables have exponents that are non-negative integers.

$$4x^5 - 7x^3 + \frac{2}{3}x^2 - \sqrt{3}$$

The degree of a polynomial is the largest exponent.

The coefficients of a polynomial are numbers in front of the variables.

The leading coefficient is the number in front of the variable with the largest exponent.

We classify polynomials in several ways:

By number of terms

Name	# of terms	Example
Monomial	1	$4x$ or -7 or x^2
Binomial	2	$4x-1$ or x^2+2
Trinomial	3	x^2+2x-1 Or $4x^5+2x^3-3x$
Polynomial	4+	$-6x^6+x^2+1+8x^4-9x^8$

By degree

Name	degree	Example
Constant	0	-8
Linear	1	$-6x-2$
Quadratic	2	$3x^2+2x$
Cubic	3	x^3
Quartic	4	$-x^4-x+1$
Quintic	5	$6x^5+4x^3+2x^2-x$

When adding or subtracting polynomials, we add like terms (those with the same variables.) We can do this vertically or horizontally.

Example:

If $f(x) = 4x^3 - 2x^2 - 5x - 4$ and

$g(x) = x^4 + 3x^2 + x - 2$

Find the following . . .

a) $f(x) + g(x)$

$x^4 + 4x^3 + x^2 - 4x - 6$

b) $g(x) - f(x)$

$x^4 - 4x^3 + 5x^2 + 6x + 2$

c) $2f(x) - 3g(x)$

$8x^3 - 4x^2 - 10x - 8 - 3x^4 - 9x^2 - 3x + 6$

$= -3x^4 + 8x^3 - 13x^2 - 13x - 2$

We can also multiply polynomials.

Example:

$$\left(2x^2+2\right)(x-4)$$

$$2x^3-8x^2+2x-8$$

Let $n(x)=2x-4$ be the number of magic math pills produced by a company at an average cost of $a(x)=-3x^3-5x^2+x$ dollars per pill, where x is the number of years since 2000. Create a function, $c(x)$, for how much money has been spent on producing these pills as a function of time, x.

$$\begin{aligned} &c(x) \\ &=\left(-3x^3-5x^2+x\right)(2x-4) \\ &=-6x^4+12x^3-10x^3+20x^2+2x^2-4x \\ &=-6x^4+2x^3+22x^2-4x \end{aligned}$$

When a polynomial is raised to a higher power, we can expand it by a routine, repetitive process. We call this Binomial Expansion.

Example:

Expand $(2x-1)^3$

$$(2x-1)(2x-1)(2x-1)+$$
$$=\left(4x^2-4x+1\right)\{2x-1\}$$
$$=8x^3-4x^2-8x^2+4x+2x-1$$
$$=8x^3-12x^2+6x-1$$

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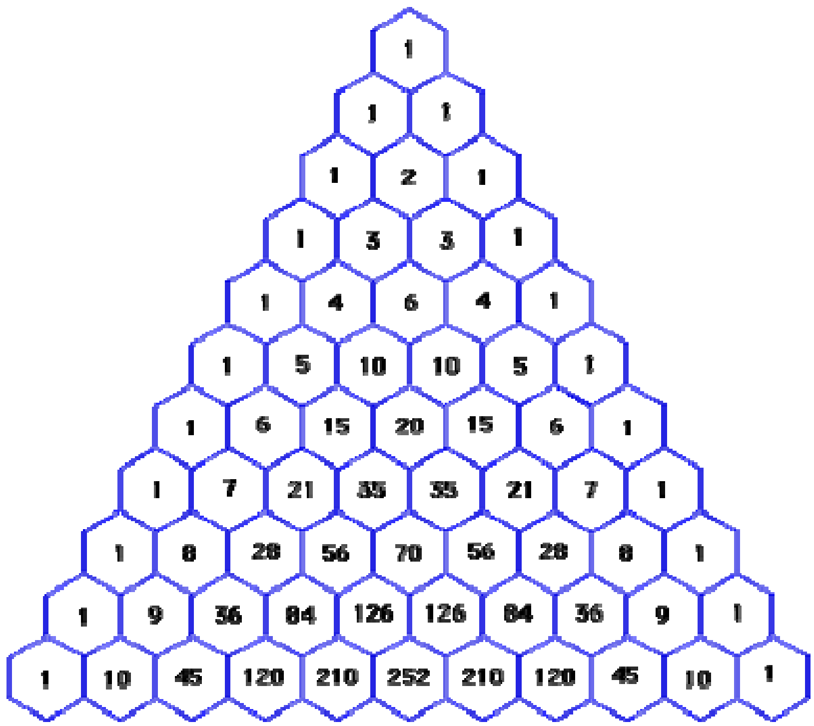
For any binomial of the form $(a+b)^n$, we can expand using a more efficient method:

Pascal's Triangle



Expression	Expansion	Triangle coeffs
$(a+b)^0$	1	1
$(a+b)^1$	$a+b$	1 1
$(a+b)^2$	$a^2+2ab+b^2$	1 2 1
$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$	1 3 3 1
$(a+b)^4$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$	1 4 6 4 1

Example:

Expand $(x-2)^4$

$$x^4(-2)^0+4x^3(-2)^1+6x^2(-2)^2+4x^1(-2)^3+x^0(-2)^4$$

$$x^4-8x^3+24x^2-32x+16$$

References:
All images TI-83+ calculator

http://mathforum.org/workshops/usi/pascal/images/pascal.hex2.gif
http://www.biografiasyvidas.com/biografia/p/fotos/pascal.jpg
http://go.hrw.com/gopages/ma/alg2_07.html

About Blaise Pascal and Pascal's Triangle
Check out: http://ptri1.tripod.com/

Pascal's Triangle was originally developed by the ancient Chinese, but Blaise Pascal was the first person to discover the importance of all of the patterns it contained. On this page, I explain how the Triangle is formed, and more importantly, many of its patterns.

He was a remarkable 17th century mathematician who made astounding contributions to many fields. He purportedly built one of the earliest calculators called the Pascaline (the computer language Pascal is named after him), built the first barometer (the Pascal, Pa, a unit of atmospheric pressure is named after him). He also, almost single handedly invented the mathematical branch of probability theory, invented the roulette wheel, and wore the first wrist watch (with he invented.) Later in life, he devoted himself to philosophy and theology, and his famous "Wager" gives us a compelling reason to believe in a divine creator. Alas, like many great geniuses, he died early, at the age of 39 of fatigue, actually wearing himself out from studying too hard. He is famous for the saying, "The heart has its reasons which reason knows not of."