

## Déjà Vu, It's Algebra 2! Lesson 14 Polynomials: Addition, Subtraction, \& Multiplication

A polynomial is an expression that consists of adding or subtracting a combination of numbers and variables. The variables have exponents that are non-negative integers.

$$
4 x^{5}-7 x^{3}+\frac{2}{3} x^{2}-\sqrt{3}
$$

The degree of a polynomial is the largest exponent.
The coefficients of a polynomial are numbers in front of the variables.

The leading coefficient is the number in front of the variable with the largest exponent.

We classify polynomials in several ways:
By number of terms

| Name | \# of terms | Example |
| :---: | :---: | :---: |
| Monomial |  | $4 x$ or -7 or $x^{2}$ |
| Binomial |  | $4 x-1$ or $x^{2}+2$ |
| Trinomial |  | $x^{2}+2 x-1$ |
|  |  | $4 x^{5}+2 x^{3}-3 x$ |
| Polynomial |  | $-6 x^{6}+x^{2}+1+8 x^{4}-9 x^{8}$ |

By degree

| Name | degree | Example |
| :---: | :---: | :---: |
| Constant |  | -8 |
| Linear |  | $-6 x-2$ |
| Quadratic |  | $3 x^{2}+2 x$ |
| Cubic |  | $x^{3}$ |
| Quartic |  | $-x^{4}-x+1$ |
| Quintic |  | $6 x^{5}+4 x^{3}+2 x^{2}-x$ |

When adding or subtracting polynomials, we add like terms (those with the same variables.) We can do this vertically or horizontally.

## Example:

If $f(x)=4 x^{3}-2 x^{2}-5 x-4$ and $g(x)=x^{4}+3 x^{2}+x-2$

Find the following . . .
a) $f(x)+g(x)$
b) $g(x)-f(x)$
c) $2 f(x)-3 g(x)$

## We can also multiply polynomials.

Example:
$\left(2 x^{2}+2\right)(x-4)$

Let $n(x)=2 x-4$ be the number of magic math pills produced by a company at an average cost of $a(x)=-3 x^{3}-5 x^{2}+x$ dollars per pill, where $x$ is the number of years since 2000. Create a function, $c(x)$, for how much money has been spent on producing these pills as a function of time, $x$.

When a polynomial is raised to a higher power, we can expand it by a routine, repetitive process. We call this Binomial Expansion.

## Example:

Expand $(2 x-1)^{3}$

## Déjà RE-Vu

For any binomial of the form $(a+b)^{n}$, we can expand using a more efficient method:

Pascal's Triangle


| Expression | Expansion | Triangle coeffs |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $(a+b)^{0}$ | 1 |  | 1 |  |  |  |
| $(a+b)^{1}$ | $a+b$ |  | 1 | 1 |  |  |
| $(a+b)^{2}$ | $a^{2}+2 a b+b^{2}$ |  | 2 | 2 | 1 |  |
| $(a+b)^{3}$ | $a^{3}+3 a^{2} b+3 a b^{2}+b^{3}$ | 1 | 3 | 3 | 1 |  |
| $(a+b)^{4}$ | $a^{4}+4 a^{3} b+6 a^{2} b^{2}+4 a b^{3}+b^{4}$ | 1 | 4 | 6 | 4 | 1 |

## Example:

Expand $(x-2)^{4}$

## References:

## All images Tl-83+ calculator

http://mathforum.org/workshops/usi/pascal/images/pascal.hex2.gif http://www.biografiasyvidas.com/biografia/p/fotos/pascal.jpg
http://go.hrw.com/gopages/ma/alg2_07.html

