Déjà Vu, It's Algebra 2!

Lesson 16

Polynomials: Finding Complex & Irrational Roots

Irrational Conjugate Theorem:

In a polynomial $P(x)$ with rational coefficients, if $a + \sqrt{b}$ is a root, then so is $a - \sqrt{b}$, provided that a is rational and $\sqrt{b}$ is irrational.

Example:

Find the exact value of all roots of

$$f(x) = 2x^3 - 9x^2 + 2 \text{ if } x = \frac{1}{2} \text{ and } x = 2 + \sqrt{6} \text{ are}$$

roots. What is its factorization?

The theorem tells us that $2 - \sqrt{6}$ is also a root. This gives us three roots for a degree three polynomial, so we know by the Fundamental Theorem of Algebra that we have found them all.

By the factor theorem, we know its factorization:

$$f(x) = 2\left(x - \frac{1}{2}\right)\left(x - 2 - \sqrt{6}\right)\left(x - 2 + \sqrt{6}\right)$$

Another closely related theorem is the . . .

Complex Conjugate Theorem:

In a polynomial $P(x)$ with real coefficients, if $a + bi$ is a root, then so is $a - bi$, where a, b are real numbers and i is the imaginary unit.

Example:

Write a polynomial $h(x)$ of lowest degree whose roots are $x = -i + 2$, $x = -\sqrt{3}$, and $x = 4$.

We know that the degree mus be 5, since the two conjugate theorems each give us an additional root. Remember that you take the plus and minus of either the i term or the radical term. It's more than just changing the middle sign.

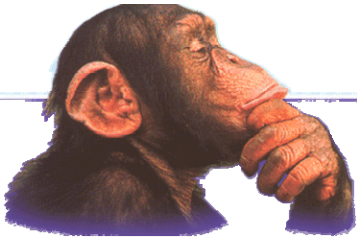The roots are: $x = 2 - i$, $2 + i$, $-\sqrt{3}$, $\sqrt{3}$, and 4

$$h(x) = (x - 2 + i)(x - 2 - i)(x + \sqrt{3})(x - \sqrt{3})(x - 4)$$

$$h(x) = \left(x^2 - 2x - ix - 2x + 4 + 2i + ix - 2i - i^2\right)\left(x^2 - 3\right)(x - 4) \quad \text{***remember } i^2 = -1$$

$$h(x) = \left(x^2 - 4x + 5\right)\left(x^2 - 3\right)(x - 4)$$

It is MUCH easier when expanding, to expand conjugate factors with each other first. You'll soon see the radicals and/or the imaginary units going "bye bye!"

Rational Root theorem:

If a polynomial has rational roots, then they will be of the form $\dfrac{p}{q}$, where p is a factor of the CONSTANT, and q is a factor of the LEADING COEFFICIENT.

Example:

List the possible rational roots, then find all complex roots of $m(x) = 2x^4 + 3x^3 + 4x^2 + 9x - 6$

List the factors of -6: $\pm 1, \pm 2, \pm 3, \pm 6$
List the factors of 2: $\pm 1, \pm 2$

Take combinations: $\pm 1, \pm\dfrac{1}{2}, \pm 2, \pm 3, \pm\dfrac{3}{2}, \pm 6$

There are 12 possible rational roots (including negatives). Although the theorem does not GUARANTEE that any of these work, if the function HAPPENS TO HAVE A RATIONAL ROOT, the theorem tells us it will be one of these 12.

Use the graph of the function to tip you off as to which, if any, are roots. The other option is to try synthetic division on all 12 numbers (which beats trying infinite random numbers). If NONE happen to work, you know the 4 roots are either Irrational or Imaginary.

$x = \dfrac{1}{2}$, $x = -2$: synthetically divide these out to get $x^2 + 3$, then solve this quadratic to get the final two: $x = i\sqrt{3}$ and $x = -i\sqrt{3}$

Déjà RE-Vu

A grain silo is shaped like a cylinder with a cone-shaped top. The cylinder is 30 feet tall. The silo can hold 1152π cubic feet of grain at full capacity. Find the radius, *x*, of the silo.

30 ft
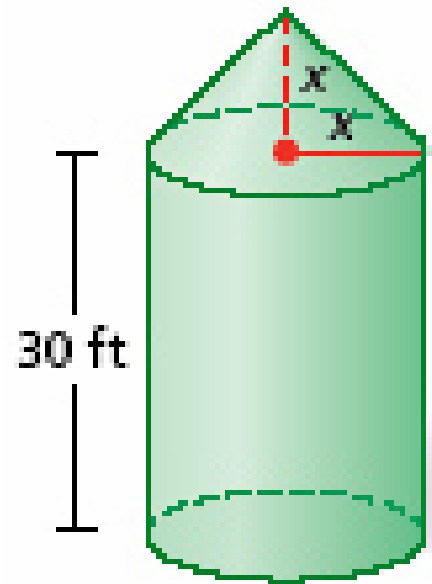
Volume of a cylinder $= \pi r^2 h$

Volume of a cone $= \dfrac{\pi}{3} r^2 h$

$$= \pi x^2 (30) + \frac{\pi}{3} x^2 (x) = 1152\pi$$

$$\frac{\pi}{3} x^3 + 30\pi x^2 = 1152\pi$$

so

Volume of the silo $\dfrac{\pi}{3} x^3 + 30\pi x^2 - 1152\pi = 0$

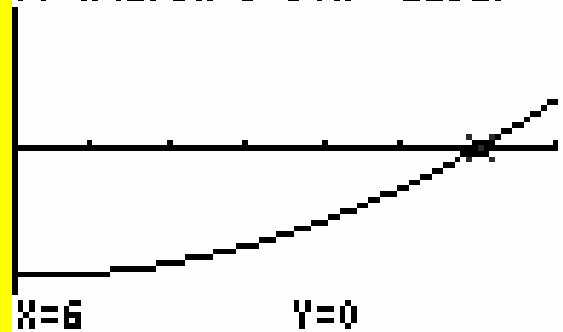Relevant *Domain* : $\{x \mid 0 < x < 6.197\}$

from calculator :

x=6 *feet*

$x = -6.431$ and $x = -89.569$ are both irrational roots in the domain of the function, but not in the relevant domain of our problem!!!!

Y1=π(1/3X^3+30X2-1152)

X=6   Y=0

WINDOW
 Xmin=0
 Xmax=7
 Xscl=1
 Ymin=-5000
 Ymax=5000
 Yscl=0
 Xres=■

References:
All images TI-83+ calculator

http://go.hrw.com/gopages/ma/alg2_07.html
http://cres1.lancs.ac.uk/~esasb1/recipes/pics/grain.gif
http://www.comunications.it/img/rational.gif