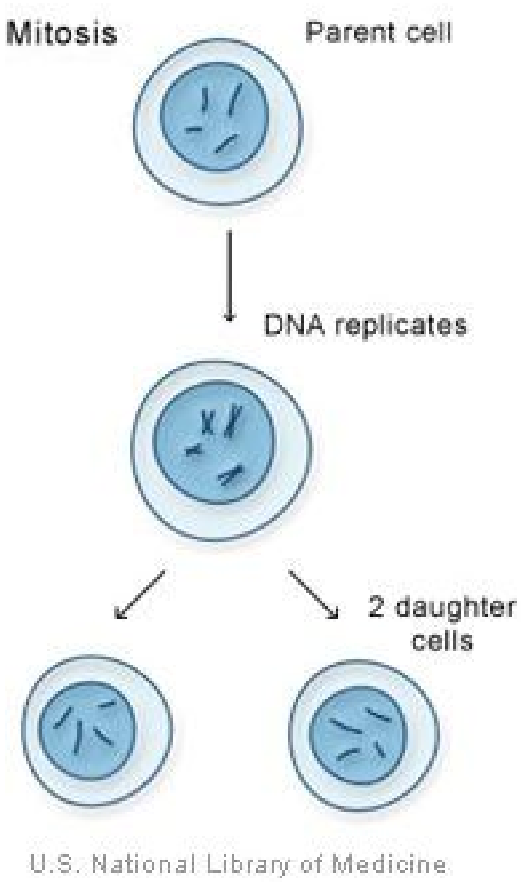
Déjà Vu, It's Algebra 2!

Lesson 17

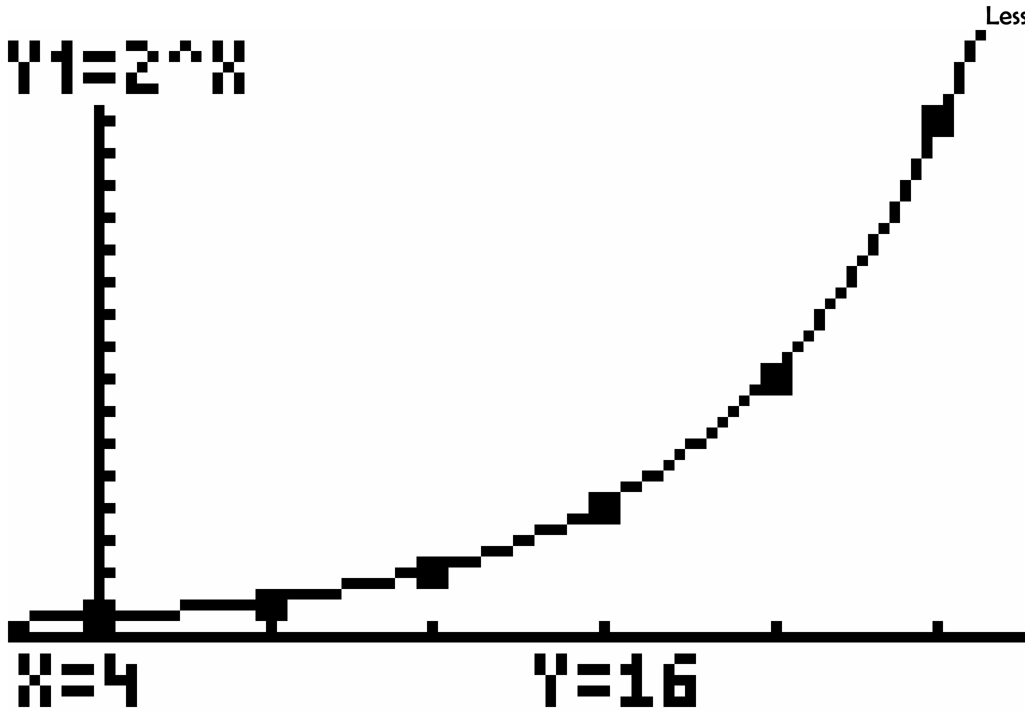
Exponential Functions

Think of cell division (mitosis and cytokinesis.)



Each new cell can produce two new ones. Assuming they all duplicate simultaneously, the population of cells DOUBLES with each new division.

Number of divisions, x	**Population**, y
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
24	$16{,}777{,}216 = 2^{24}$
x	$y = 2^x$



Because the population (y-values) increase by a constant ratio (or factor), we say the population grows exponentially. The equation $y = 2^x$ is an example of an exponential growth function.

Notice that the growth rate is proportional to the population at any given time. That is, the bigger the population, the faster it grows.

In general, the form for a parent exponential function is

$$f(x) = a \cdot b^x$$

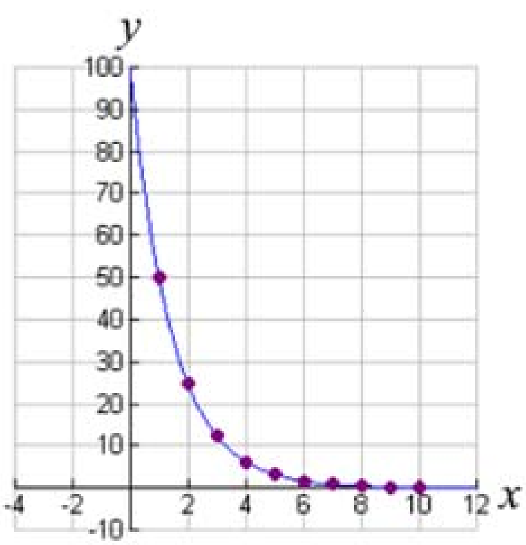
a is the initial value or y-intercept
b is the BASE, $b > 0$, $b \neq 1$
x is the EXPONENT (the variable is the exponent!!)

Observe how the graphs of exponential functions change based upon the values of *a* and *b:*

$y = a \bullet b^x$

Example: $y = 100(0.5)^x$

when $a > 0$ and the b is between 0 and 1, the graph will be decreasing (decaying).
For this example, each time x is increased by 1, y decreases to one half of its previous value.

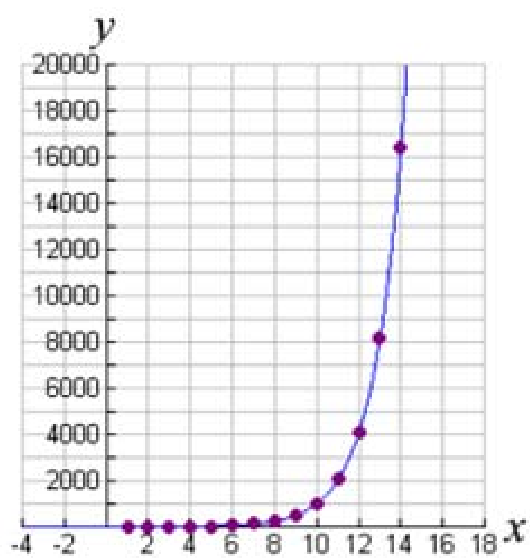


Such a situation is called **Exponential Decay.**

$y = a \bullet b^x$

Example: $y = 1(2)^x$

when $a > 0$ and the b is greater than 1, the graph will be increasing (growing).
For this example, each time x is increased by 1, y increases by a factor of 2.

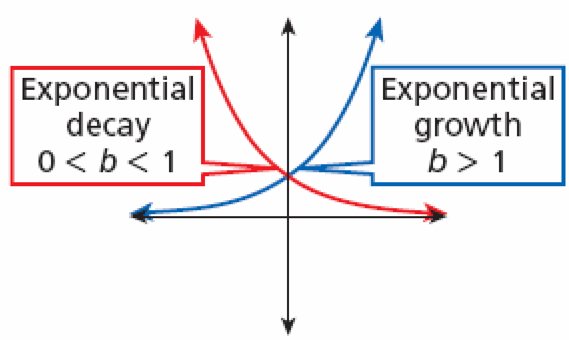


Such a situation is called **Exponential Growth.**

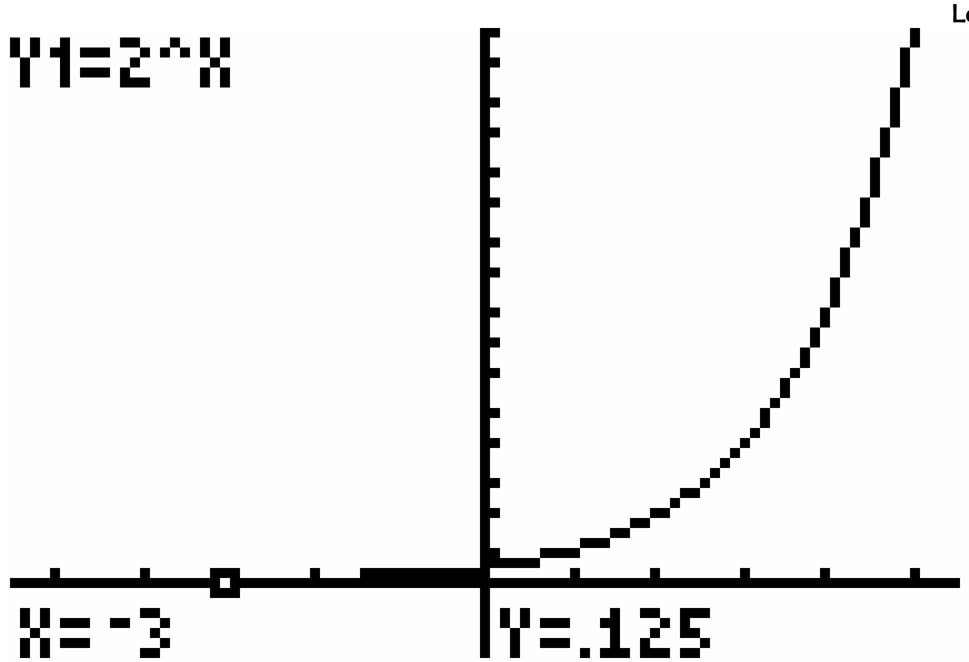
Exponential growth and decay can be used to model many things in the real world, such as cell division, population studies, money earning interest, appreciation or depreciation of material objects, and radioactive substances.

Any quantity that grows or decays by a fixed percent, fixed ratio, or fixed factor at regular intervals exhibits either **exponential growth** or **exponential decay**.

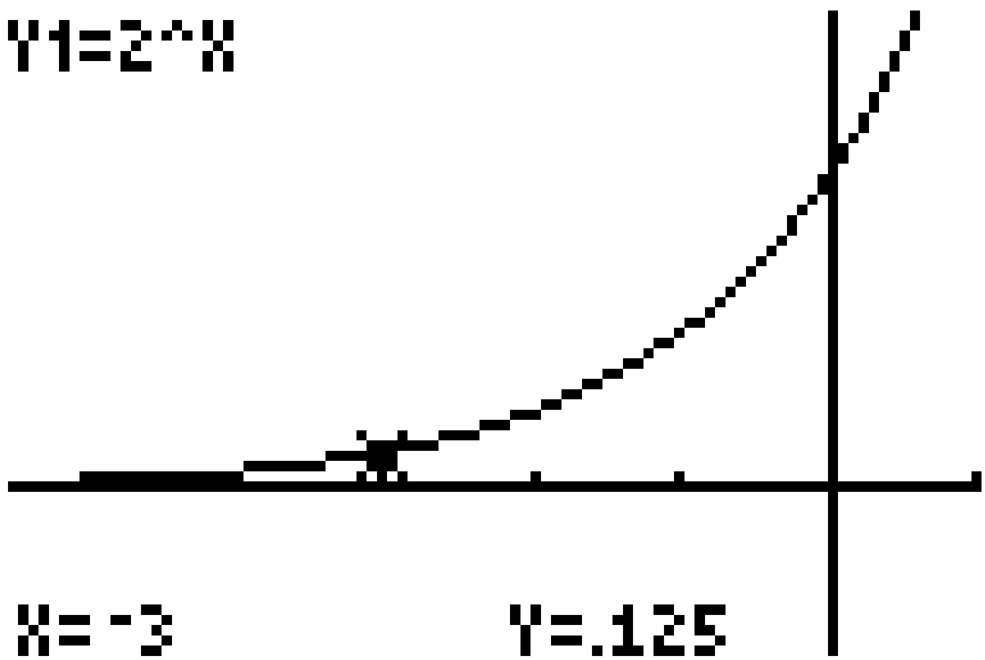
Parent Exponential Function $f(x) = b^x$, where $b > 0$ and $b \neq 1$



Exponential functions have two different end behaviors. In one case, they increase without bound and in another case, they are bounded below by a Horizontal Asymptote at $y = 0$



The graph below is an expanded view of the same point in a zoomed window.



Usually, when working with exponential functions, we are looking at time as our independent variable, so we use t instead of x. Also, because we are usually discussing populations, or money, or some specific quantity, we use capital A for "amount" instead of y.

$$A(t) = a \cdot b^t$$

Example:

In 1626, the Dutch bought Manhattan Island, now part of New York City, from the Algonquin Indians for \$24 worth of merchandise. Suppose that, instead, \$24 had been invested in an account that paid 3.5% interest each year, what would the balance be in 2008?

Number of years, t, after 1626	Amount, A, in dollars, after t years
0	24
1	$24+24(.035)=24(1.035)=24.84$
2	$24(1.035)(1.035)=24(1.035)^2=25.71$
3	$24(1.035)^2(1.035)=24(1.035)^3=26.61$
4	$24(1.035)^4=27.54$
5	$24(1.035)^5=28.50$
t	$A(t)=24(1.035)^t$

The year 2008 corresponds to a t value of $2008 - 1626 = 382$

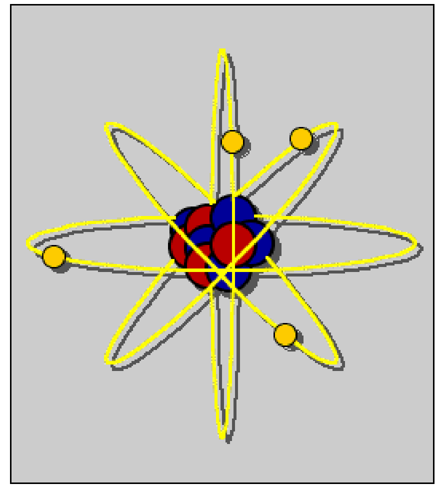
$$A(382) = 24\left(1.035\right)^{382} = 12229955.10$$

That means that if the Dutch had invested their $24, today, it would be worth $12,229,995.10. That's over 12 million dollars! Exponential growth really demonstrates the time-value of money.

Déjà RE-Vu

CARBON DATING:

The half-life of C^{14}, a radioactive carbon isotope, is about 5730 years. ***(Half-life is the amount of time it takes for half of the amount of a substance to decay.)***



If a piece of ancient charcoal contains only 15% as much of the radioactive carbon as a piece of modern charcoal, how long ago was the tree burned to make the ancient charcoal?

We first need to create a model: $A(t) = a\left(b^t\right)$

Because we are dealing with percents, we can choose a convenient original value of 100grams of carbon-14. So the artifact now has 15 grams (100grams times 15%) or carbon-14.

We can now assume that $a = 100$.

We need another value to find our base, *b*. This is based on the half-life: $(t, A) = (5730, 50)$

So,

$A(t) = 100\left(b^t\right) \to 50 = 100\left(b^{5730}\right) \to b \approx 0.9998790392$

so

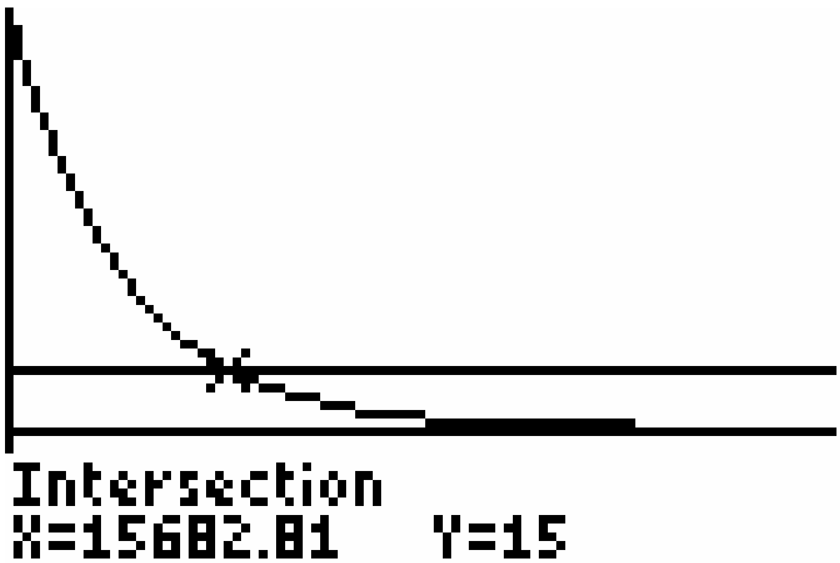
$A(t) = 100\left(0.9998790392^t\right)$

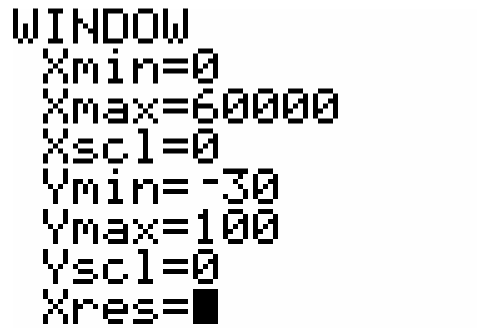
If the current sample contains only 15 grams, that is $A(t)$, and we wish to solve for *t*.

So

$15 = 100\left(0.9998790392^t\right)$

From the calculator, we see the tree was burned about 15,683 years ago!


Intersection
X=15682.81 Y=15


WINDOW
Xmin=0
Xmax=60000
Xscl=0
Ymin=-30
Ymax=100
Yscl=0
Xres=

References:

All images TI-83+ calculator or TI-Interactive Software

http://www.daviddarling.info/images/cell_division.jpg
http://www.studioglyphic.com/images/blog/purchase-of-manhattan-print.jpg
http://go.hrw.com/gopages/ma/alg2_07.html