Déjà Vu, It's Algebra 2!

Lesson 18

Inverse and Logarithmic Functions

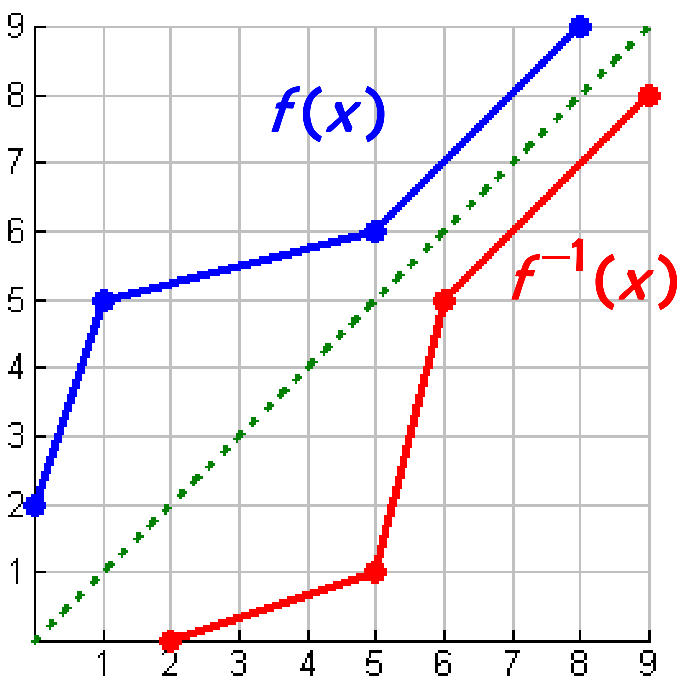
A function $y = f(x)$ is defined by the ordered pairs listed in the following table.

$f(x)$

x	0	1	5	8
y	2	5	6	9

$f^{-1}(x)$

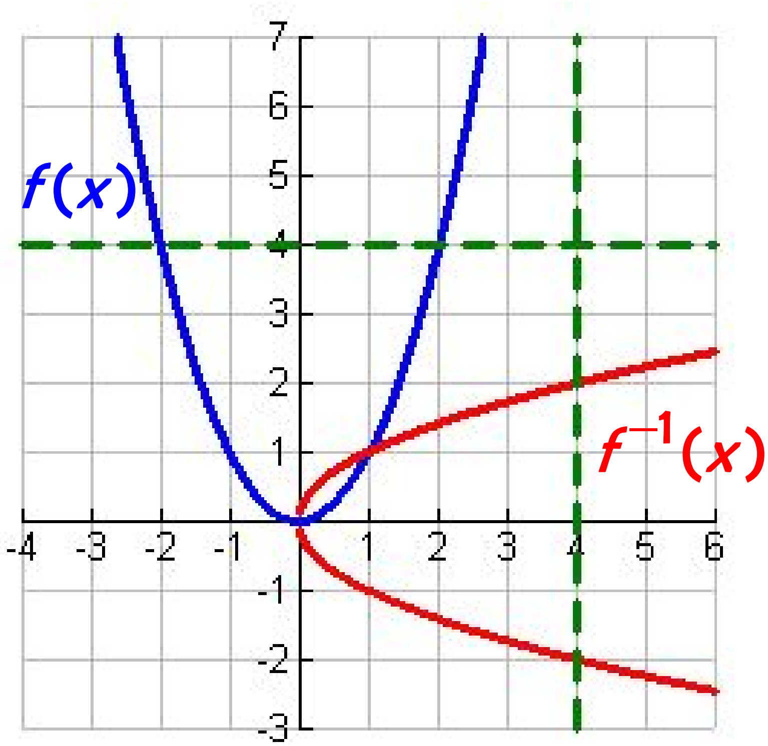
x	2	5	6	9
y	0	1	5	8



$f(x)$	$f^{-1}(x)$
D: $[0,8]$	D: $[2,9]$
R: $[2,9]$	R: $[0,8]$

Summary regarding inverse functions:

- All x and y values interchange
- The Domain and Range interchange
- The x-axis and y-axis interchange
- Inverse functions are reflections across the line $y = x$
- Because a vertical line becomes a horizontal line when reflected across $y = x$, an inverse will pass the vertical line test for functions if and only if the function passes the horizontal line test! Such functions are called one-to-one. This means not all functions have inverses that are functions!!



- Algebraically, you can find an equation of an inverse by interchanging the x and y values, then resolve for y.

Example:

Find the inverse function $f^{-1}(x)$ for the function $f(x)=3(x-5)$, then verify by graphing.

$y=3(x-5)$ replace $f(x)$ with y

$x=3(y-5)$ interchange x and y

$x=3y-15$

$3y=x+15$

$y=\frac{x+15}{3}$

$y=\frac{1}{3}x+5$

$f^{-1}(x)=\frac{1}{3}x+5$

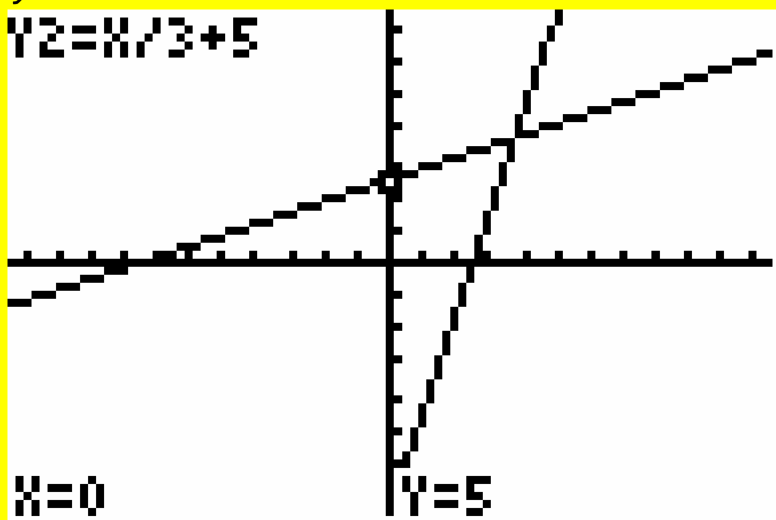
Check algebraically:

$f(4)=-3$

$f^{-1}(-3)=4$

It's easier to graph the function if you expand it into slope-intercept form of a line:

$y=3x-15$

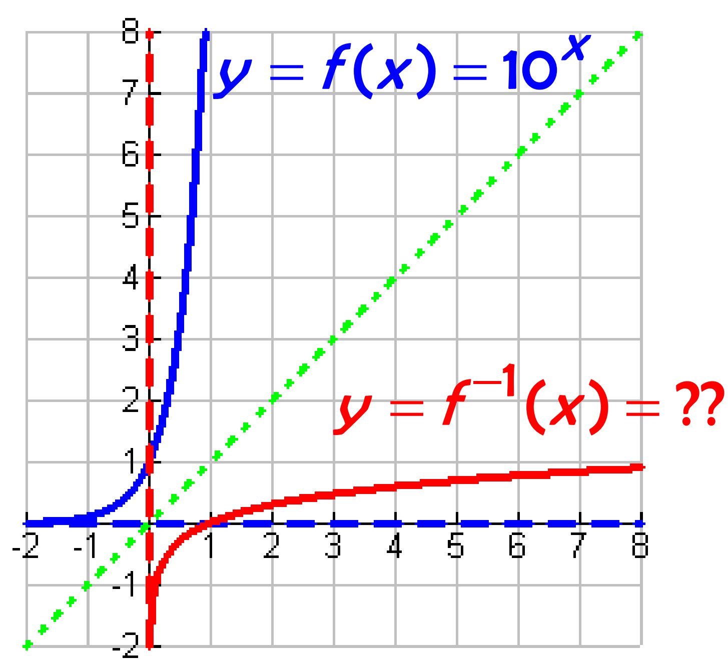


Example:

Find the inverse of the exponential function $y=10^x$.

$x=10^y$

We are already algebraically stuck at this point. We have not learned any method for removing the y from the exponent. It's time to learn how to do that!!!!



A Logarithm (or Log for short) is the exponent to which a specified base is raised to obtain a given value.

Example:

Find the value of x in each of the following.

a) $2^x = 32$

$32 = 2^5$
So $x = 5$
5 is the log, base 2, of 32
Or
$\log_2 32 = 5$

b) $10^x = 10,000$

$10000 = 10^4$
So $x = 4$
4 is the log, base 10, of 10000
Or
$\log_{10} 10,000 = 4$

c) $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

$\frac{1}{27} = \left(\frac{1}{3}\right)^3$
So $x = 3$
3 is the log, base $\frac{1}{3}$, of $\frac{1}{27}$ Or
$\log_{1/3} \frac{1}{27} = 3$

Here's a very important Theorem which will allow us to convert between log and exponential equations:

$$y = b^x \Leftrightarrow \log_b y = x$$

$$b > 0, b \neq 1$$

Log equation	Exponential equation
$\log_2 64 = 6$	$2^6 = 64$
$\log_7 7 = 1$	$7^1 = 7$
$\log_3 1 = 0$	$3^0 = 1$
$\log_5 0.04 = -2$	$5^{-2} = \frac{1}{25} = 0.04$
$\log_3 81 = x$	$3^x = 81$
$\log_4 4^x = x$	$4^x = 4^x$
$\log_8 x = \log_8 x$	$8^{\log_8 x} = x$

Basic properties of logs:

1. $\log_b 1 = 0$
2. $\log_b b^x = x$
3. $b^{\log_b x} = x$

Déjà RE-Vu

Coding a message:

The following message was coded with the following exponential function $f(x) = 2^x$

$$[8192; 2; 1048576; 256][512; 524288][64; 2097152; 16384]$$

If x corresponds to a letter in the alphabet, and $f(x)$ is the transformed value, decipher the message.

We must first find the inverse function, which we know will be a function, since exponential functions are one-to-one. We do this by using the conversion theorem to get $f^{-1}(x) = \log_2 x$. We then plug all the values into this function for x, find the function value, then find what letter that number corresponds to in the alphabet. A table helps organize the information.

x	$f^{-1}(x) = \log_2 x$	Letter of Alphabet
8192	13	M
2	1	A
1048576	20	T
256	8	H
512	9	I
524288	19	S
64	6	F
2097152	21	U
16384	14	N

MATH IS FUN

References:

All images TI-83+ calculator or TI-Interactive Software

http://www.gilwellmississauga.org/upcoming_events.html
http://blog.wired.com/photos/uncategorized/smiley_face.jpg