Déjà Vu, It's Algebra 2!

Lesson 19

Properties of Logarithms

Remember this very important Theorem. Because logarithms and exponentiation are INVERSE operations of each other, we can convert between log and exponential equations:

$$\boxed{y = b^x \Leftrightarrow \log_b y = x}$$

$$b > 0, b \neq 1$$

<u>Two usual choices for our base, *b*.</u>

- 10
  - known as the common base
  - $\log_{10} x = \log x$, the common log
  - Found on the calculator
- $e \approx 2.718281828\ldots$
  - known as the natural base
  - $\log_e x = \ln x$, the natural log
  - Found on the calculator

The Four Basic Properties of Logs

1. $\log_b x + \log_b y = \log_b xy$

$$\underset{\text{expand}}{\overset{\text{condense}}{\rightleftarrows}}$$

Example:

$\log 10 + \log 1000 =$

$\log 10 + \log 1000 = \log(10 \cdot 1000)$

$\log_{10} 10^1 + \log_{10} 10^3 = \log 10000$

$1 + 3 = \log_{10} 10^4$

$4 = 4$

Because

$\log 10000 = \log(10^1 \cdot 10^3) = \log(10^{1+3}) = \log_{10} 10^4 = 4$

2. $\log_b x - \log_b y = \log_b \left(\dfrac{x}{y}\right)$

$$\underset{\text{expand}}{\overset{\text{condense}}{\rightleftarrows}}$$

Example:

$\log_2 32 - \log_2 4 =$

$\log_2 32 - \log_2 4 = \log_2 \left(\dfrac{32}{4}\right)$

$\log_2 2^5 - \log_2 2^2 = \log_2 8$

$5 - 2 = \log_2 2^3$

$3 = 3$

Because $\log 10000 = \log_2 \left(\dfrac{32}{4}\right) = \log_2 \left(\dfrac{2^5}{2^2}\right) = \log_2 2^{5-2} = \log_2 2^3 = 3$

3. $\log x^n = n \log x$

Example:

$\ln\left(2^3\right) =$

expand $\longrightarrow$

$\longleftarrow$ condense

$\ln\left(2^3\right) = 3\ln 2$

$\ln 8 = 3\left(0.693147\cdots\right)$

$2.07944\ldots = 2.07944\ldots$

```
3ln(2)
          2.079441542
ln(8)
          2.079441542
```

4. The Change of Base formula

$$\log_b x = \frac{\log_a x}{\log_a b}, a > 1, a \neq 0$$

Example:

$\log_9 27 =$

$\log_9 27 =$

$\frac{\log_3 27}{\log_3 9} = \frac{\log_3 3^3}{\log_3 3^2} = \frac{3}{2} = 1.5$

or

$\frac{\ln 27}{\ln 9} = 1.5$

or

$\frac{\log 27}{\log 9} = 1.5$

```
ln(27)/ln(9)
              1.5
log(27)/log(9)
              1.5
```

```
ln(27)
          3.295836866
log(27)
          1.431363764
```

Example:

Expand the following logarithmic expression:

$$\ln\left(\frac{3x^2}{2y^3z^4}\right) =$$

$$\ln 3 + 2\ln x - \ln 2 - 3\ln y - 4\ln z$$

Example:

Condense the following logarithmic expression:

$$2\log(3x) - 3\log y + \log 2 + 2\log x - 4\log z =$$

$$\log(3x)^2 - \log y^3 + \log 2 + \log x^2 - \log z^4$$

$$= \log\left(\frac{(9x^2)(2)(x^2)}{(y^3)(z^4)}\right)$$

$$= \log\left(\frac{18x^4}{y^3z^4}\right)$$

Summary

Properties of logs

Expand → ← **Condense**

1. $\log_b xyz = \log_b x + \log_b y + \log_b z$

2. $\log_b \dfrac{x}{yz} = \log_b x - \log_b y - \log_b z$

3. $\log_b x^n = n\log_b x$

(Bases MUST be the same to expand/condense)

To change to any base of choice:

4. $\log_b x = \dfrac{\log_a x}{\log_a b}$

Common Errors:

1. $\log(x+y) \neq \log x + \log y$

$\log(10+100) = \log 110 = 2.041$
$\neq \log 10 + \log 100 = 1 + 2 = 3$

2. $\log(xy) \neq (\log x)(\log y)$

$\log(10 \cdot 100) = \log 1000 = 3$
$\neq \log 10 \cdot \log 100 = 1 \cdot 2 = 2$

3. $\log\left(\dfrac{x}{y}\right) \neq \dfrac{\log x}{\log y}$

$\log\left(\dfrac{100}{10}\right) = \log 10 = 1$
$\neq \dfrac{\log 100}{\log 10} = \dfrac{2}{1} = 2$

Déjà RE-Vu

Seismologists use the Richter scale to express the energy, or magnitude, of an earthquake. The Richter magnitude of an earthquake, M, is related to the energy released in ergs E shown by the formula

$$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

Because the Richter scale is logarithmic, an increase of 1 corresponds to a release of 10 times as much energy. An increase of 2 is $10^2 = 100$ times stronger

The tsunami that devastated parts of Asia in December 2004 was spawned by an earthquake with magnitude 9.3.

How many times as much energy did this earthquake release compared to the 6.9-magnitude earthquake that struck San Francisco in 1989?

We need to find the E for both magnitudes.
For Asian earthquake:

$$9.3 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

$$13.95 = \log\left(\frac{E}{10^{11.8}}\right)$$

$$10^{13.95} = \frac{E}{10^{11.8}}$$

$$E = \left(10^{13.95}\right)\left(10^{11.8}\right)$$

$$E = 10^{25.75} \approx 5.623 \times 10^{25}\, ergs$$

For San Francisco earthquake:

$$6.9 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

$$10.35 = \log\left(\frac{E}{10^{11.8}}\right)$$

$$10^{10.35} = \frac{E}{10^{11.8}}$$

$$E = \left(10^{10.35}\right)\left(10^{11.8}\right)$$

$$E = 10^{22.15} \approx 1.413 \times 10^{22}\, ergs$$

To figure out how many time stronger the Asian earthquake is, we divide the two values, which is the same as subtracting their exponents.

$$\frac{10^{25.75}}{10^{22.15}} = 10^{25.75-22.15} = 10^{3.6} \approx 3981$$

So the earthquake that spawned the tsunami was almost 4000 times stronger that than the SF earthquake!!!!

References:

All images TI-83+ calculator or TI-Interactive Software
http://www.lgchronicle.net/files/earthquake.jpg