Déjà Vu, It's Algebra 2!

Lesson 19

Properties of Logarithms

Remember this very important Theorem. Because logarithms and exponentiation are INVERSE operations of each other, we can convert between log and exponential equations:

$$y = b^x \Leftrightarrow \log_b y = x$$

$$b > 0, b \neq 1$$

Two usual choices for our base, *b*.

- 10
  - known as the common base
  - $\log_{10} x = \log x$, the common log
  - Found on the calculator
- $e \approx 2.718281828\ldots$
  - known as the natural base
  - $\log_e x = \ln x$, the natural log
  - Found on the calculator

The Four Basic Properties of Logs

1. $\log_b x + \log_b y = \log_b xy$

condense ⟶ ⟵ expand

Example:

$\log 10 + \log 1000 =$

2. $\log_b x - \log_b y = \log_b \left(\frac{x}{y} \right)$

condense ⟶ ⟵ expand

Example:

$\log_2 32 - \log_2 4 =$

3. $\log x^n = n \log x$

Example:

$\ln\left(2^3\right) =$

expand ⟶

⟵ condense

```
3ln(2)
        2.079441542
ln(8)
        2.079441542
```

4. The Change of Base formula

$$\log_b x = \frac{\log_a x}{\log_a b}, a > 1, a \neq 0$$

Example:

$\log_9 27 =$

```
ln(27)/ln(9)
                1.5
log(27)/log(9)
                1.5
```

```
ln(27)
        3.295836866
log(27)
        1.431363764
```

Example:

Expand the following logarithmic expression:

$$\ln\left(\frac{3x^2}{2y^3z^4}\right) =$$

Example:

Condense the following logarithmic expression:

$$2\log(3x) - 3\log y + \log 2 + 2\log x - 4\log z =$$

Summary

Properties of logs

Expand → ← Condense

1. $\log_b xyz = \log_b x + \log_b y + \log_b z$
2. $\log_b \frac{x}{yz} = \log_b x - \log_b y - \log_b z$
3. $\log_b x^n = n \log_b x$

(Bases MUST be the same to expand/condense)

To change to any base of choice:

4. $\log_b x = \frac{\log_a x}{\log_a b}$

Common Errors:

1. $\log(x + y) \neq \log x + \log y$

2. $\log(xy) \neq (\log x)(\log y)$

3. $\log\left(\frac{x}{y}\right) \neq \frac{\log x}{\log y}$

Déjà RE-Vu

Seismologists use the Richter scale to express the energy, or magnitude, of an earthquake. The Richter magnitude of an earthquake, M, is related to the energy released in ergs E shown by the formula

$$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

Because the Richter scale is logarithmic, an increase of 1 corresponds to a release of 10 times as much energy. An increase of 2 is $10^2 = 100$ times stronger

The tsunami that devastated parts of Asia in December 2004 was spawned by an earthquake with magnitude 9.3.

How many times as much energy did this earthquake release compared to the 6.9-magnitude earthquake that struck San Francisco in 1989?

References:

All images TI-83+ calculator or TI-Interactive Software

http://www.lgchronicle.net/files/earthquake.jpg