Déjà Vu, It's Algebra 2!

Lesson 20

Exponential & Log Equations

An Exponential Equation is an equation containing one or more expressions that have a variable as an exponent.

We will look at two methods for solving exponential equations:

1. Try to get the bases the same.

   If $b^x = b^y$, then $x = y$ $(b > 0, b \neq 1)$

2. Take the logarithm of each side.

   If $a = b$, then $\log a = \log b$ $(a > 0, b > 0)$

Example:

Solve:

$$9^{8-x} = 27^{x-3}$$

$$\left(3^2\right)^{8-x} = \left(3^3\right)^{x-3}$$

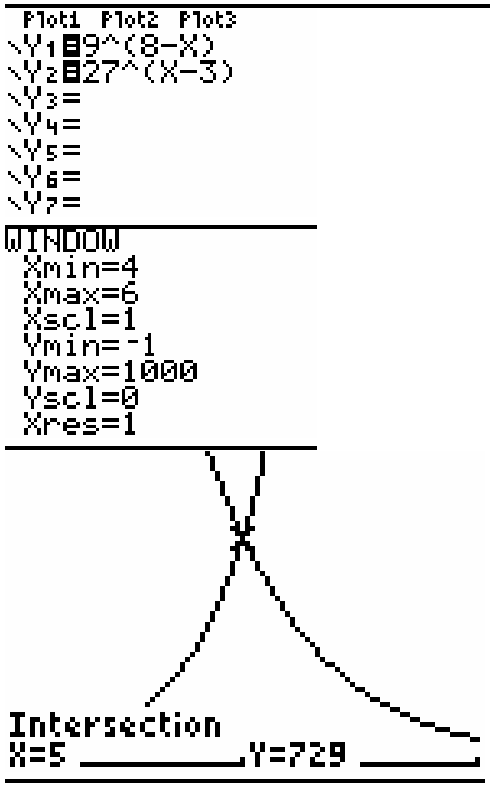
$$3^{16-2x} = 3^{3x-9}$$

$$16-2x = 3x-9$$

$$-5x = -25$$

$$x = 5$$

Check on calculator:



Example:

Solve:

$$5\left(\frac{1}{32}\right)^{2x-1} = 40\left(4^{4-2x}\right)$$

$$\left(32^{-1}\right)^{2x-1} = 8\left(\left(2^2\right)^{4-2x}\right)$$

$$\left(2^5\right)^{-2x+1} = 2^3\left(2^{8-4x}\right)$$

$$2^{-10x+5} = 2^{3+8-4x}$$

$$-10x+5 = 11-4x$$

$$-6x = 6$$

$$x = -1$$

Example:

Solve:

$$4^{x-1} = 5$$

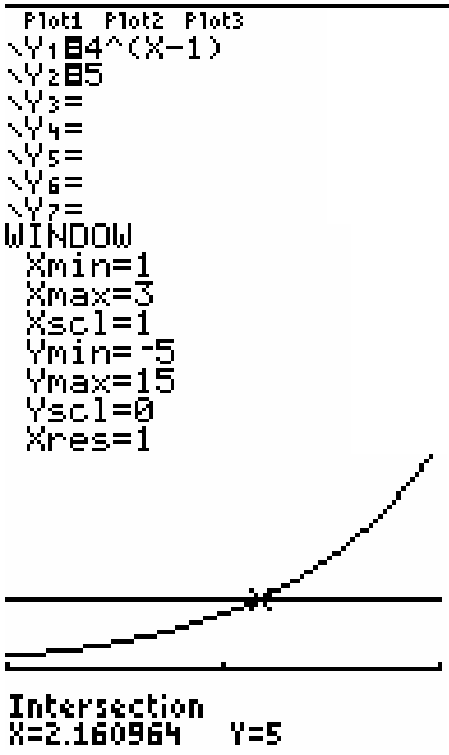
$$\log\left(4^{x-1}\right) = \log(5)$$

$$(x-1)\log 4 = \log 5$$

$$x - 1 = \frac{\log 5}{\log 4}$$

$$x = \frac{\log 5}{\log 4} + 1 \approx 2.161$$

Check on calculator:



Example:

Solve:

$$6e^{-x} = 5\left(2^{2x}\right)$$

$$\ln\left(6e^{-x}\right) = \ln\left(5 \cdot 2^{2x}\right)$$

$$\ln 6 + \ln e^{-x} = \ln 5 + \ln 2^{2x}$$

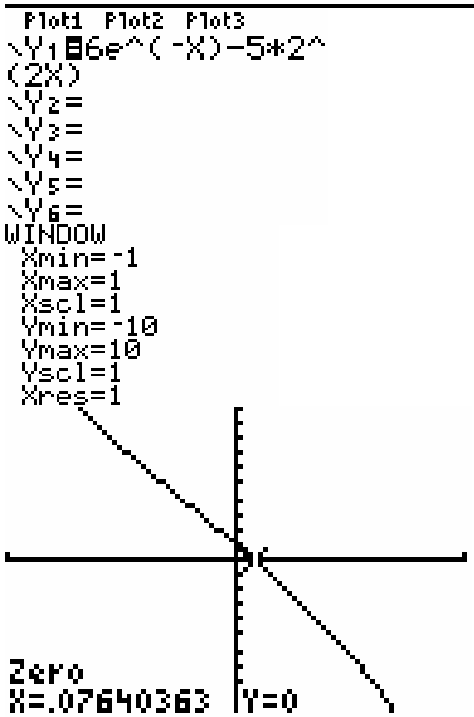
$$\ln 6 - x = \ln 5 + 2x \ln 2$$

$$-x - 2x \ln 2 = \ln 5 - \ln 6$$

$$x(-1 - 2\ln 2) = \ln 5 - \ln 6$$

$$x = \frac{\ln 5 - \ln 6}{-1 - 2\ln 2} = \frac{\ln 6 - \ln 5}{\ln 4 + 1} \approx 0.0764$$

Check on calculator:



A logarithmic equation is an equation with a logarithmic expression that contains a variable.

You can solve a logarithmic equation by doing the following:

1. Isolate the logarithm (this may require condensing!!)

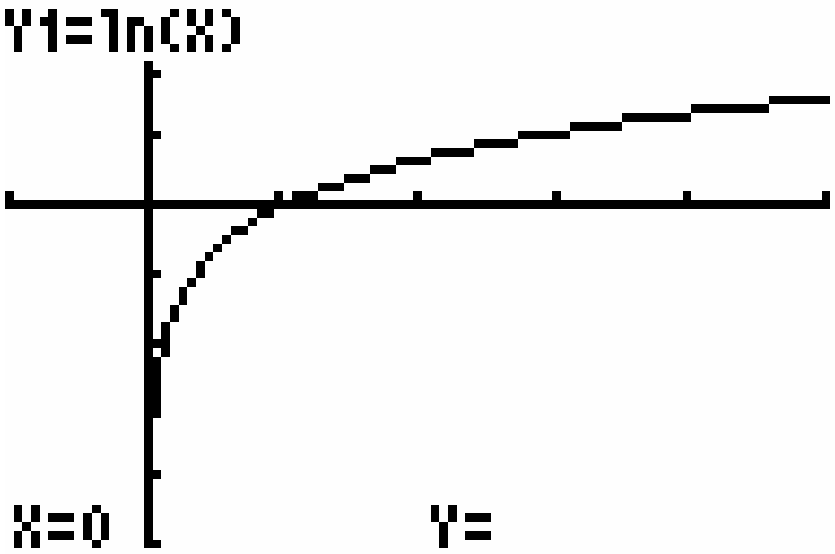
$$\log_b x = a$$

2. Convert it to exponential and solve

$$\log_b x = a$$

$$b^a = x$$

3. Check your solutions: (remember, we can only take logs of POSITIVE numbers!!)



Example:

Solve:

$$2\log_6(2x-1) = -2$$

$\log_6(2x-1) = -1$

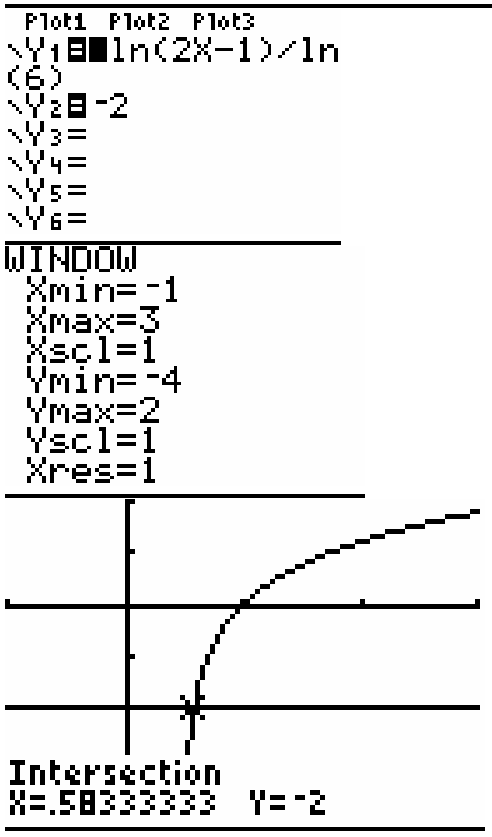
$6^{-1} = 2x-1$

$2x = 1+\frac{1}{6}$

$2x = \frac{7}{6}$

$x = \frac{7}{12} \approx 0.5833$

Check on calculator:



Example:

Solve:

$$\log_{12} x + \log_{12}(x+1) = 1$$

$\log_{12}(x(x+1)) = 1$

$12^1 = x^2 + x$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, 3$

$x = 3$ ($x = -4$ does not work)

Example:

Solve:

$$\log_4 x^2 = 7$$

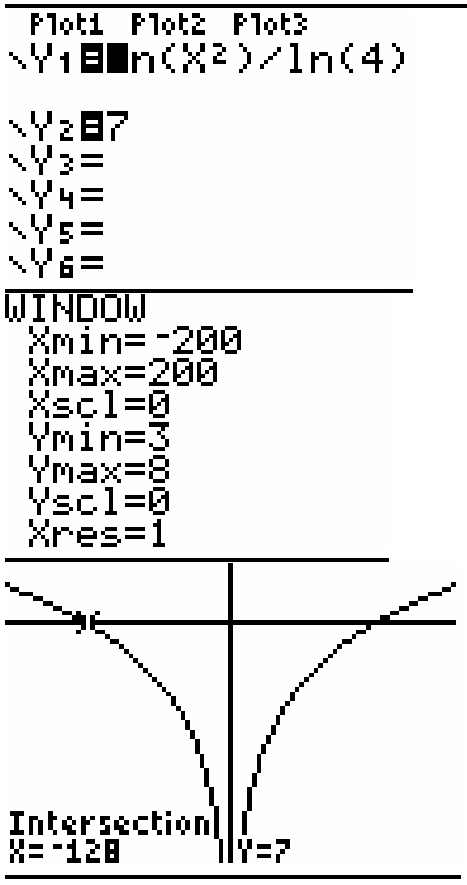
$4^7 = x^2$

$x^2 = 16384$

$x = \pm\sqrt{16384}$

$x = \pm 128$

Check on calculator:



Example:

Solve:

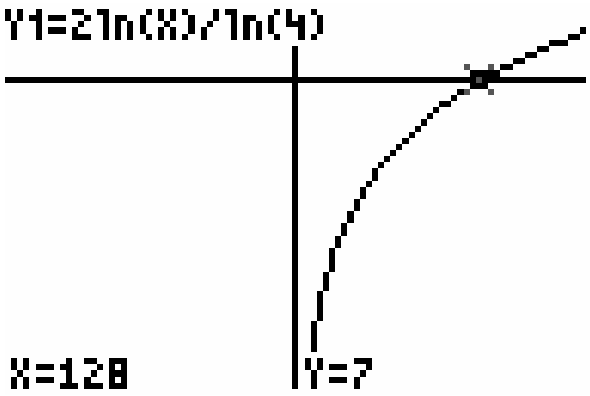
$$\log_4 x^2 = 7$$

$2\log_4 x = 7$

$\log_4 x = \frac{7}{2}$

$4^{7/2} = x$

$x = \sqrt{4^7} = 128$



Déjà RE-Vu

Interesting varieties:

Example:

Solve:

$$\log_2\left(\log_3 x\right) = -1$$

$2^{-1} = \log_3 x$

$\log_3 x = \frac{1}{2}$

$3^{1/2} = x$

$x = \sqrt{3} \approx 1.732$

Example:

Solve:

$$\ln(x+5) = e^{x-5}$$

$\ln(x+5) = e^{x-5}$

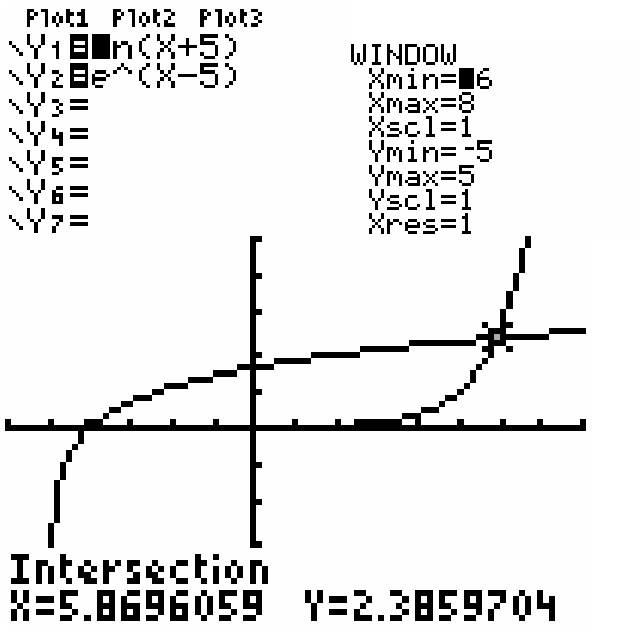
$x + 5 = e^{e^{x-5}}$

or

$\ln(\ln(x+5)) = x - 5$

Can't isolate the *x*. Any attempts to "undo" the other side "unlocks" one *x*, but "traps" the other.

We solve this one on the calculator.



References:

All images TI-83+ calculator or TI-Interactive Software