



Déjà Vu, It's Algebra 2!

Lesson 20

Exponential & Log Equations

An **Exponential Equation** is an equation containing one or more expressions that have a variable as an exponent.

We will look at two methods for **solving** exponential equations:

1. Try to get the bases the same.

$$\text{If } b^x = b^y, \text{ then } x = y \quad (b > 0, b \neq 1)$$

2. Take the logarithm of each side.

$$\text{If } a = b, \text{ then } \log a = \log b \quad (a > 0, b > 0)$$

Example:**Solve:**

$$9^{8-x} = 27^{x-3}$$

$$(3^2)^{8-x} = (3^3)^{x-3}$$

$$3^{16-2x} = 3^{3x-9}$$

$$16 - 2x = 3x - 9$$

$$-5x = -25$$

$$x = 5$$

Example:**Solve:**

$$5\left(\frac{1}{32}\right)^{2x-1} = 40\left(4^{4-2x}\right)$$

$$(32^{-1})^{2x-1} = 8\left((2^2)^{4-2x}\right)$$

$$(2^5)^{-2x+1} = 2^3(2^{8-4x})$$

$$2^{-10x+5} = 2^{3+8-4x}$$

$$-10x + 5 = 11 - 4x$$

$$-6x = 6$$

$$x = -1$$

Check on calculator:

```

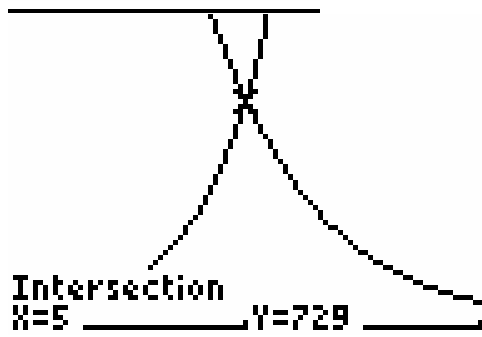
Plot1 Plot2 Plot3
Y1=9^(8-X)
Y2=27^(X-3)
Y3=
Y4=
Y5=
Y6=
Y7=

```

```

WINDOW
Xmin=4
Xmax=6
Xscl=1
Ymin=-1
Ymax=1000
Yscl=0
Xres=1

```



Example:**Solve:**

$$4^{x-1} = 5$$

$$\log(4^{x-1}) = \log(5)$$

$$(x-1)\log 4 = \log 5$$

$$x-1 = \frac{\log 5}{\log 4}$$

$$x = \frac{\log 5}{\log 4} + 1 \approx 2.161$$

Example:**Solve:**

$$6e^{-x} = 5(2^{2x})$$

$$\ln(6e^{-x}) = \ln(5 \cdot 2^{2x})$$

$$\ln 6 + \ln e^{-x} = \ln 5 + \ln 2^{2x}$$

$$\ln 6 - x = \ln 5 + 2x \ln 2$$

$$-x - 2x \ln 2 = \ln 5 - \ln 6$$

$$x(-1 - 2 \ln 2) = \ln 5 - \ln 6$$

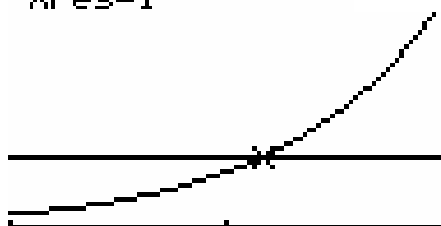
$$x = \frac{\ln 5 - \ln 6}{-1 - 2 \ln 2} = \frac{\ln 6 - \ln 5}{\ln 4 + 1} \approx 0.0764$$

Check on calculator:

```

Plot1 Plot2 Plot3
Y1=4^(X-1)
Y2=5
Y3=
Y4=
Y5=
Y6=
Y7=
WINDOW
Xmin=1
Xmax=3
Xscl=1
Ymin=-5
Ymax=15
Yscl=0
Xres=1

```



```

Intersection
X=2.160964 Y=5

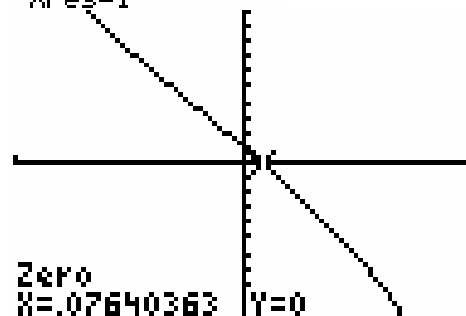
```

Check on calculator:

```

Plot1 Plot2 Plot3
Y1=6e^(-X)-5*2^
(2X)
Y2=
Y3=
Y4=
Y5=
Y6=
WINDOW
Xmin=-1
Xmax=1
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```



```

Zero
X=.07640363 Y=0

```

A logarithmic equation is an equation with a logarithmic expression that contains a variable.

You can solve a logarithmic equation by doing the following:

1. **Isolate the logarithm** (this may require condensing!!)

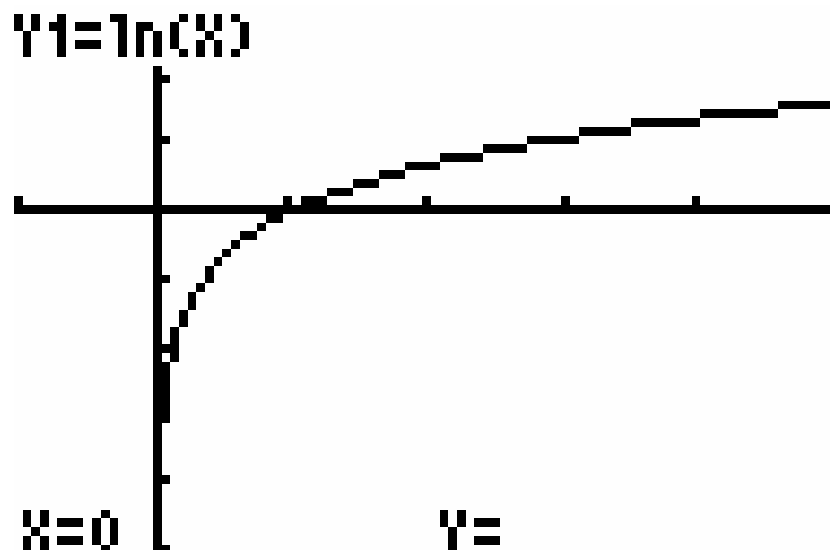
$$\log_b x = a$$

2. **Convert it** to exponential and solve

$$\log_b x = a$$

$$b^a = x$$

3. **Check your solutions:** (remember, we can only take logs of POSITIVE numbers!!)



Example:

Solve:

$$2\log_6(2x-1) = -2$$

$$\log_6(2x-1) = -1$$

$$6^{-1} = 2x-1$$

$$2x = 1 + \frac{1}{6}$$

$$2x = \frac{7}{6}$$

$$x = \frac{7}{12} \approx 0.5833$$

Example:

Solve:

$$\log_{12} x + \log_{12}(x+1) = 1$$

$$\log_{12}(x(x+1)) = 1$$

$$12^1 = x^2 + x$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

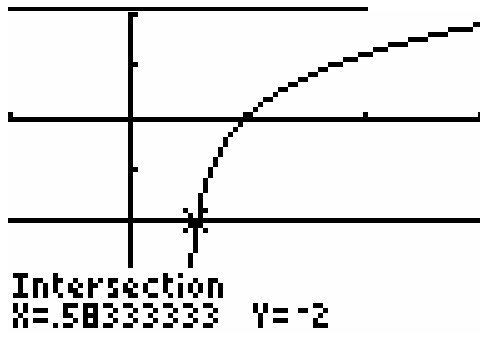
$$x = -4, 3$$

$$x = 3 \text{ (} x = -4 \text{ does not work)}$$

Check on calculator:

```
Plot1 Plot2 Plot3
\Y1=ln(2X-1)/ln
(6)
\Y2=-2
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
Xmin=-1
Xmax=3
Xscl=1
Ymin=-4
Ymax=2
Yscl=1
Xres=1
```



Example:**Solve:**

$$\log_4 x^2 = 7$$

$$4^7 = x^2$$

$$x^2 = 16384$$

$$x = \pm\sqrt{16384}$$

$$x = \pm 128$$

Example:**Solve:**

$$\log_4 x^2 = 7$$

$$2\log_4 x = 7$$

$$\log_4 x = \frac{7}{2}$$

$$4^{7/2} = x$$

$$x = \sqrt{4^7} = 128$$

Check on calculator:

```

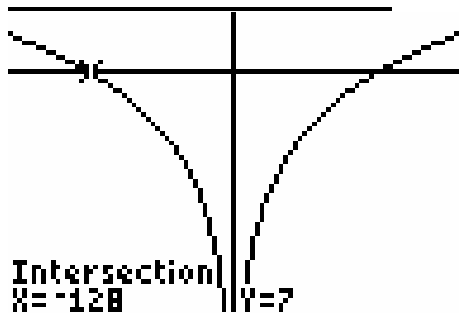
Plot1 Plot2 Plot3
Y1=ln(X^2)/ln(4)
Y2=7
Y3=
Y4=
Y5=
Y6=

```

```

WINDOW
Xmin=-200
Xmax=200
Xscl=100
Ymin=-10
Ymax=10
Yscl=1
Xres=1

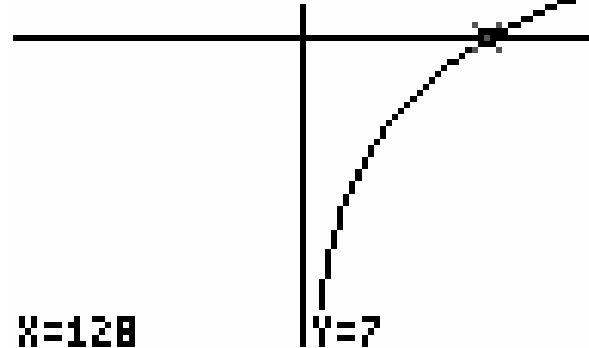
```



```

Y1=2*ln(X)/ln(4)

```



Déjà RE-Vu

Interesting varieties:

Example:

Solve:

$$\log_2(\log_3 x) = -1$$

$$2^{-1} = \log_3 x$$

$$\log_3 x = \frac{1}{2}$$

$$3^{1/2} = x$$

$$x = \sqrt{3} \approx 1.732$$

Example:

Solve:

$$\ln(x + 5) = e^{x-5}$$

$$\ln(x + 5) = e^{x-5}$$

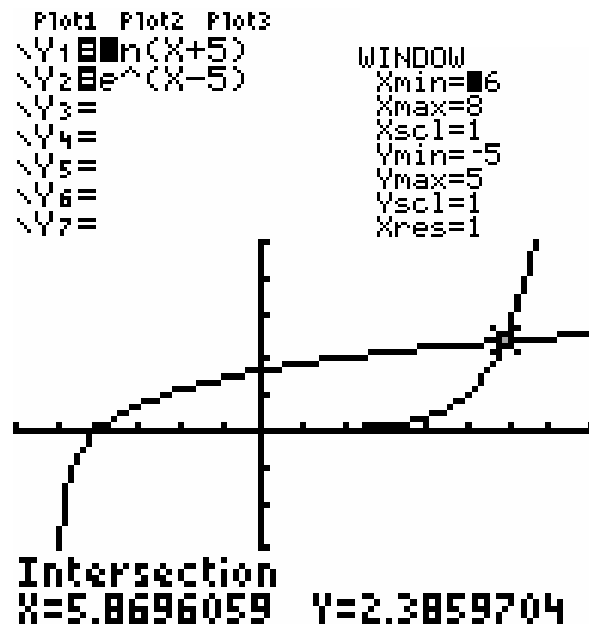
$$x + 5 = e^{e^{x-5}}$$

or

$$\ln(\ln(x + 5)) = x - 5$$

Can't isolate the x . Any attempts to "undo" the other side "unlocks" one x , but "traps" the other.

We solve this one on the calculator.



References:

All images TI-83+ calculator or TI-Interactive Software

