Déjà Vu, It's Algebra 2!

Lesson 21

Rational Expressions: Operations & Equations

A rational number is a quotient of two integers

Ex) $\frac{4}{6}$, $-\frac{1}{5}$, 1.5, 0.3333..., 7

Notice the key word "ratio" in "rational."

A rational expression is a quotient of two polynomials

Ex) $\frac{x+3}{x^2+x-2}$, $\frac{4x^5}{6x^7}$, $\frac{4}{3}x$, x^5-x, 8

The following are NOT rational expressions. Why not?

Ex) $\frac{\sqrt{x}}{3x+1}$, $\frac{4x^3+3x^2}{2^x-2x}$, $\frac{\ln x}{x}$

Has a radical term, which is not a polynomial

Has an exponential term, which is not a polynomial

Has an natural log term, which is not a polynomial

Just as we did with rational numbers, we can simplify rational expressions by dividing out common factors!!!!! (we cannot divide out common terms!)

Review:

A Factor is a number or variable that is multiplied or divided to another number or variable in an expression.

A Term is an expression that is added or subtracted to another expression.

Example:

$$\frac{66}{63} = \frac{60+6}{60+3} \text{ " = " } \frac{60+6}{60+3} = \frac{6}{3} = 2$$

$\frac{66}{63} \neq 2$, so the operation is illegitimate, so don't do it. You CANNOT divide out common terms!!!!

Example:

Simplify the following rational expressions.

a) $\frac{5}{15} + \frac{x^5}{x^2} + \frac{6x}{4x^3} + \frac{2+x}{7+x}$

$\frac{1}{3} + x^3 + \frac{3}{2x^2} + \frac{2+x}{7+x}$

b) $\frac{x^2+x-12}{x^2+6x+8}$

$\frac{(x+4)(x-3)}{(x+4)(x+2)} = \frac{x-3}{x+2}$

Because we now have ratios of polynomials, we likely have denominators with variables in them. Certain values of the variables MIGHT make the denominator zero. These values will make the expression undefined. WE MUST FIND ANY SUCH VALUES IN THE *ORIGINAL* EXPRESSION.

Example:

Find the values for which the rational expressions are undefined, then simplify.

a) $\frac{2x}{x^2}$

$x^2 \neq 0$

$so\ x \neq 0$

$\frac{2x}{x^2}$

$= \frac{2}{x},\ x \neq 0$

b) $\frac{x^2 - 3x - 10}{2x^2 + 5x + 2}$

We will factor the numerator and denominator first.

$\frac{(x+2)(x-5)}{(x+2)(2x+1)}$

$= \frac{x-5}{2x+1},\ x \neq -2,\ -\frac{1}{2}$

c) $\frac{x-2}{2-x}$

It almost looks like these immediately divide out, but careful, they don't. We can rewrite the denominator by factoring out a negative one.

$\frac{x-2}{-(-2+x)}$

$= -\frac{x-2}{x-2}$

$= -1,\ x \neq 2$

As you can see, simplifying rational expressions requires a lot of factoring of quadratics. If you need help on this topic, ask your teacher, or go back to that section in your textbook and practice.

We may also multiply and divide rational expressions in the same fashion we did with rational numbers.

Examples:

Perform the indicated operation and simplify.

a) $\frac{3x^5y^3}{2x^3y^7} \bullet \frac{10x^3y^4}{9x^2y^5}$

$\frac{5x^3}{3y^5}$

b) $\frac{x-3}{4x+20} \bullet \frac{x+5}{x^2-9}$

$\frac{1}{4(x+3)} = \frac{1}{4x+12}$

c) $\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5}$

$\frac{x^2y^3}{3}$

d) $\frac{x^4-9x^2}{x^2-4x+3} \div \frac{x^4+2x^3-8x^2}{x^2-16}$

$\frac{(x+3)(x-4)}{(x-1)(x-2)}$

Déjà RE-Vu

We can review our new skills in a new context:
Solving rational equations!!!

Examples:

Solve

a) $\frac{x^2 - 25}{x - 5} = 14$

$x = 9$

b) $\frac{x^2 + 3x - 10}{x - 2} = 7$

no solution