Déjà Vu, It's Algebra 2!

Lesson 22

Rational Expressions: Addition/Subtraction & Complex Fractions

Recall how to combine rational numbers:

$$\frac{3}{2}+\frac{7}{3}-\frac{1}{6}$$

$$\frac{3}{2}\left(\frac{3}{3}\right)+\frac{7}{3}\left(\frac{2}{2}\right)-\frac{1}{6}$$

$$\frac{9+14-1}{6}=\frac{22}{6}=\frac{11}{3}$$

The same process applies when adding or subtracting rational expressions.

$$\frac{2}{x+1}+\frac{x}{x-1}-\frac{x^2}{x^2-1}$$

$$\frac{2}{x+1}\left(\frac{x-1}{x-1}\right)+\frac{x}{x-1}\left(\frac{x+1}{x+1}\right)-\frac{x^2}{x^2-1}$$

$$=\frac{2x-2+x^2+x-x^2}{(x-1)(x+1)}=\frac{3x-2}{x^2-1},\ x\neq\pm1$$

Example:

$$\frac{2x}{3x+1}+\frac{5}{x}-\frac{x+4}{x^2-x}$$

$$\frac{2x}{3x+1}+\frac{5}{x}-\frac{x+4}{x(x-1)}$$

$$\frac{2x}{3x+1}\left(\frac{x^2-x}{x^2-x}\right)+\frac{5}{x}\left(\frac{(3x+1)(x-1)}{(3x+1)(x-1)}\right)-\frac{x+4}{x(x-1)}\left(\frac{3x+1}{3x+1}\right)$$

$$=\frac{2x^3-2x^2+15x^2-10x-5-3x^2-13x-4}{(3x+1)(x^2-x)}$$

$$=\frac{2x^3+10x^2-23x-9}{3x^3-2x^2-x},\ x\neq-\frac{1}{3},0,1$$

Example:

$$3\left(x-y\right)^{-1}-\frac{\left(x+y\right)^{-1}}{2}$$

$$\frac{3}{x-y}-\frac{1}{2\left(x+y\right)}=\frac{3}{x-y}\left(\frac{2\left(x+y\right)}{2\left(x+y\right)}\right)-\frac{1}{2\left(x+y\right)}\left(\frac{x-y}{x-y}\right)$$

$$\frac{6x+6y-x+y}{2\left(x^2-y^2\right)}=\frac{5x+7y}{2x^2-2y^2}$$

A complex (compound) fraction is a fraction, containing another fraction in its numerator, denominator, or both. In general, an expression with a complex fraction is NOT is simplified form.

Example:

$$\frac{1+\frac{2}{x}}{5x-2}$$

Method I

$$\left(1+\frac{2}{x}\right)\div\left(\frac{5x-2}{1}\right)=\left(\frac{x+2}{x}\right)\bullet\left(\frac{1}{5x-2}\right)$$

$$=\frac{x+2}{5x^2-2x},\ x\neq 0,\frac{2}{5}$$

Method II

$$\frac{1+\frac{2}{x}}{5x-2}\left(\frac{x}{x}\right)=\frac{x+2}{5x^2-2x},\ x\neq 0,\frac{2}{5}$$

Example:

Simplify $\dfrac{\dfrac{3}{x}-\dfrac{x}{4}}{\dfrac{x-2}{x}}$

Method I

$$\left(\frac{3}{x}-\frac{x}{4}\right)\div\left(\frac{x-2}{x}\right)$$

$$\left(\frac{12-x^2}{4x}\right)\bullet\left(\frac{x}{x-2}\right)=\frac{12-x^2}{4x-8},\ x\neq 0,2$$

Method II

$$\frac{\frac{3}{x}-\frac{x}{4}}{\frac{x-2}{x}}\left(\frac{4x}{4x}\right)=\frac{12-x^2}{4x-8},\ x\neq 0,2$$

Example:

$$\frac{3x^{-1} - y^{-1}}{x^{-1} + 2y^{-1}}$$

$$\frac{\frac{3}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{2}{y}}\left(\frac{xy}{xy}\right) = \frac{3y - x}{y + 2x}$$

$$= \frac{-x + 3y}{2x + y}$$

Déjà RE-Vu

Suppose your average speed driving to San Antonio is 60 mph, but because of traffic, you only average 40 mph on the return trip. What is your average speed for the entire trip?

Let the one-way distance equal d. So the total distance traveled (round trip) is $2d$.

Using the equation Distance = Rate x Time, $d = rt$, solving for Time, t, we get $t = \frac{d}{r}$.

The Total time: $t = \frac{d}{60} + \frac{d}{40}$

Average Speed = (Total Distance) / (Total time): $\frac{2d}{\frac{d}{60} + \frac{d}{40}}$

$$\frac{2d}{\frac{d}{60} + \frac{d}{40}} \left(\frac{60 \cdot 40}{60 \cdot 40} \right) = \frac{4800d}{40d + 60d}$$

$$= \frac{4800d}{100d} = 48mph$$

References:

http://home.earthlink.net/~fliegs/images/roadtrip.gif
http://stephen.geek.nz/images/70MPH.gif