Déjà Vu, It's Algebra 2!

Lesson 23

Rational Functions

A rational function is a function that can be expressed as the quotient of two polynomials. We are interested in the graphs of such functions.

Some can be transformations of the parent function

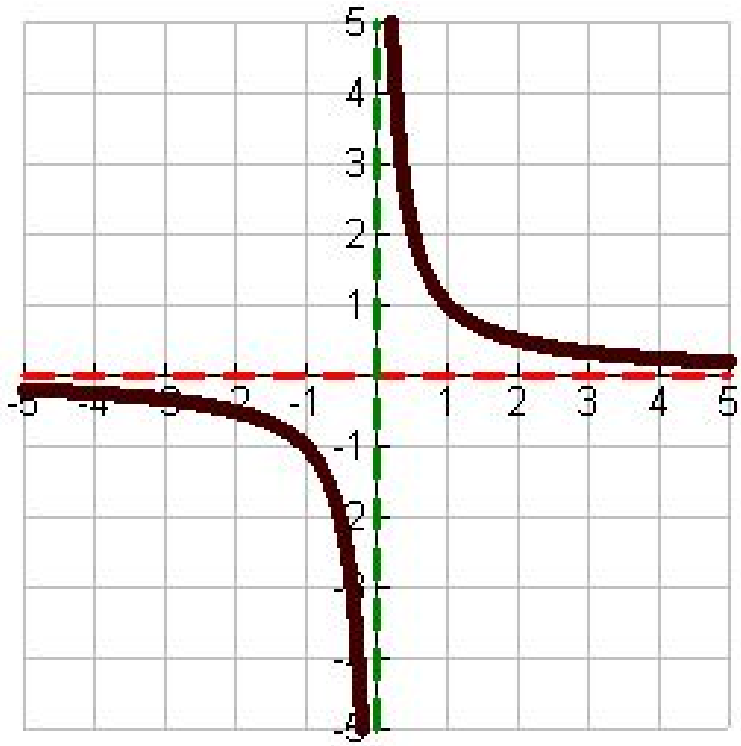
$$f(x)=\frac{1}{x},\ x\neq 0$$

Vertical Asymptote at $x=0$
Horizontal Asymptote at $y=0$

y-intercepts: none
x-intercepts: none

Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(-\infty,0)\cup(0,\infty)$

Monotonic Decreasing on
$(-\infty,0)\cup(0,\infty)$



Transformations of parent functions are HUGELY important. Remember the following form:

$$f(x) = \frac{a}{(x-c)} + d$$

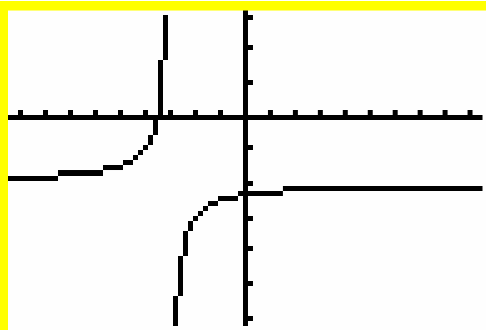
$|a|$ is a vertical dilation of $f(x)$. If $a<0$, it is a reflection across the x-axis

c is a horizontal shift. The new vertical asymptote is at $x=c$

d is a vertical shift. The new horizontal asymptote is at $y=d$

Example:

Graph $g(x) = \frac{-1}{x+3} - 2$. State the domain & range, and list the asymptotes.



Domain: $(-\infty,-3)\cup(-3,\infty)$

Range $(-\infty,-2)\cup(-2,\infty)$

Vertical Asymptote @ $x=-3$

Horizontal Asymptote @ $y=-2$

The previous graph contained a <u>discontinuity</u>, a gap or break in the graph that causes you to lift your pencil when sketching it from left to right.

Rational functions have two types of discontinuities:

1) <u>Vertical Asymptote (VA)</u>—a type of infinite discontinuity. It is a value that makes only the denominator zero.

2) <u>Removable Discontinuity (Hole)</u>—a type of point discontinuity. It is a value that makes BOTH the numerator and denominator zero.

Example:

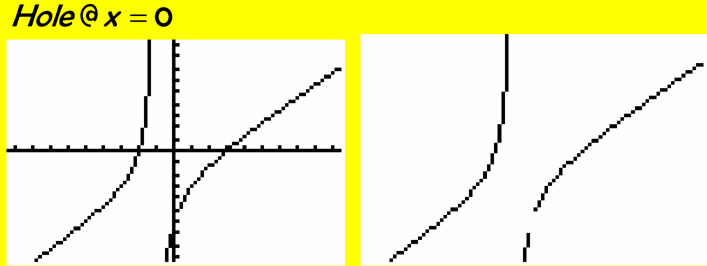
Identify the discontinuities of the following rational function. Verify graphically.

$$p(x) = \frac{(x-3)\left(x^2+2x\right)}{x^2+x}$$

$$p(x) = \frac{\cancel{x}(x-3)(x+2)}{\cancel{x}(x+1)}$$

$VA @ x = -1$

$Hole @ x = 0$



Other important features of rational functions, aside from discontinuities, we wish to find are x-intercepts and horizontal asymptotes (which are NOT discontinuities.)

The x-intercepts are the roots/zeros of the numerator (as long as they are not roots of the denominator as well!)

Example:

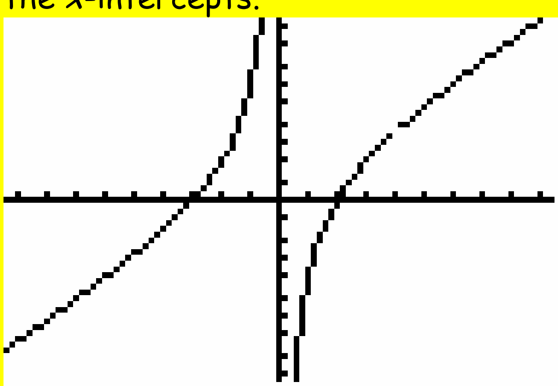
Find the x-intercepts of $q(x) = \dfrac{(x^2 + x - 6)(x - 4)}{x^2 - 4x}$

First, find the domain, that is, any discontinuity. A value cannot be an x-intercept if it is NOT in the domain of the function. Domain: $x \neq 0$.

Next, factor the numerator and denominator completely:

$$q(x) = \frac{(x+3)(x-2)(x-4)}{x(x-4)},\ x \neq 0,4$$

The only values that exclusively make the numerator zero are $x = -3, 2$. These are the x-intercepts.

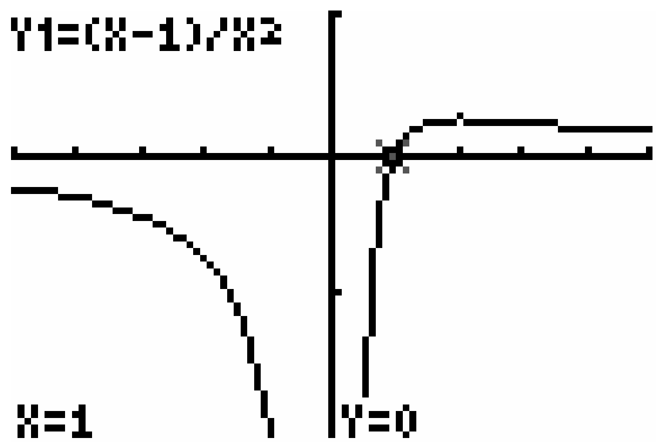


Notice the Vertical Asymptote at $x = 0$, and the hole at $x = 4$. There does not appear to be a horizontal asymptote.

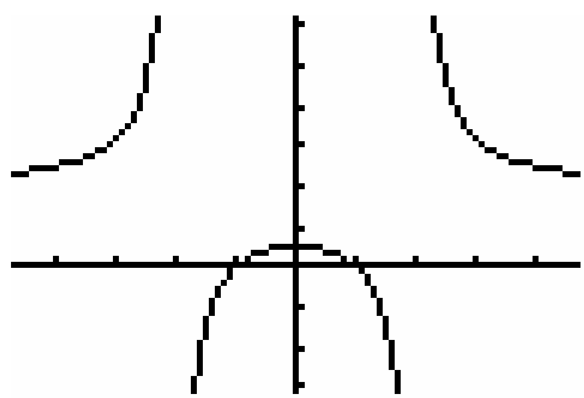
MEMORIZE THE FOLLOWING!!!!!!

There are three cases for possible horizontal asymptotes:

- If the degree of the numerator is greater than that of the denominator, there is no Horizontal Asymptote (like the previous two examples.)
- If the degree of the denominator is greater than the numerator, there is a Horizontal Asymptote at $y = 0$
  - $f(x) = \dfrac{x-1}{x^2}$



- If the degrees of the numerator and denominator are the same, there is a Horizontal Asymptote at $y =$ the quotient of the leading coefficients
  - $f(x) = \dfrac{2x^2-2}{x^2-4}$



Déjà RE-Vu

The high school band is planning a trip to play at a college bowl game. The trip will cost **\$500** per band member plus a **\$2000** deposit for the whole group.

a) Write a function to represent the average cost of the trip per band member.

b) Graph the function. Identify a Relevant domain and range, *x*- or *y*-intercepts, discontinuities, and horizontal asymptotes.

c) Find the total cost per person if 50 band members attend the bowl game.

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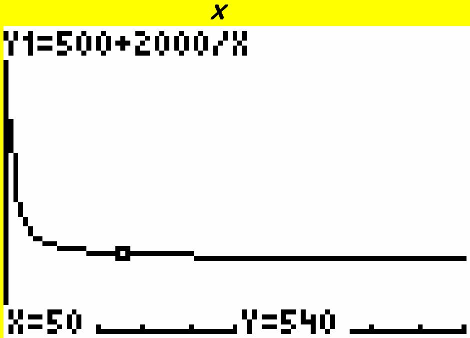
a) $c(x) = 500 + \frac{2000}{x}$, x is the number of students, $c(x)$ is the average cost per student.



b) Domain: $x > 0$. Range: $y > 500$ VA@ $x = 0$. HA@ $y = 500$.
No intercepts.

c) $c(50) = 500 + \frac{2000}{50} = 500 + 40 = \$540/\textit{person}$

References:

http://logos.simpleplants.com/Schools-Education/thumbs/Schools-Classroom-Activities-Marching_Band_2.gif