Déjà Vu, It's Algebra 2!

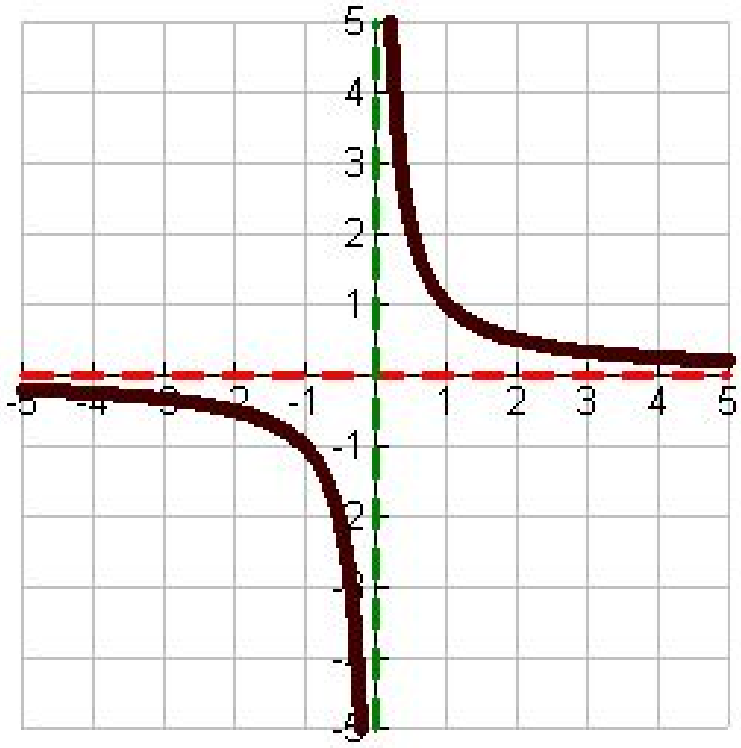
Lesson 23

Rational Functions

A rational function is a function that can be expressed as the quotient of two polynomials. We are interested in the graphs of such functions.

Some can be transformations of the parent function

$$f(x) = \frac{1}{x},\ x \neq 0$$



Transformations of parent functions are HUGELY important. Remember the following form:

$$f(x) = \frac{a}{(x-c)} + d$$

Example:

Graph $g(x) = \frac{-1}{x+3} - 2$. State the domain & range, and list the asymptotes.

The previous graph contained a discontinuity, a gap or break in the graph that causes you to lift your pencil when sketching it from left to right.

Rational functions have two types of discontinuities:

1) Vertical Asymptote (VA)—a type of infinite discontinuity. It is a value that makes only the denominator zero.

2) Removable Discontinuity (Hole)—a type of point discontinuity. It is a value that makes BOTH the numerator and denominator zero.

Example:

Identify the discontinuities of the following rational function. Verify graphically.

$$p(x) = \frac{(x-3)(x^2+2x)}{x^2+x}$$

Other important features of rational functions, aside from discontinuities, we wish to find are *x*-intercepts and horizontal asymptotes (which are NOT discontinuities.)

The *x*-intercepts are the roots/zeros of the numerator (as long as they are not roots of the denominator as well!)

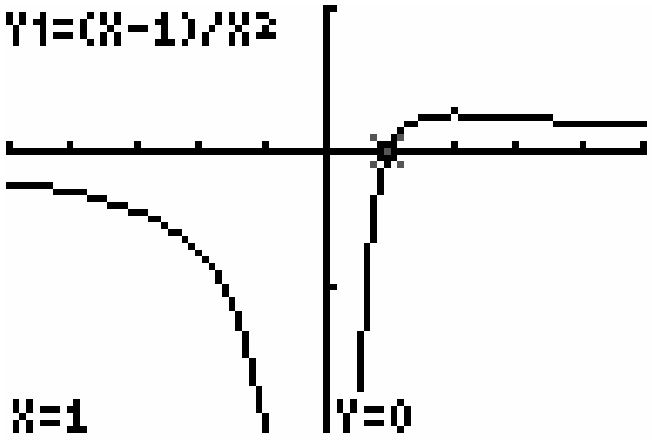
Example:

Find the *x*-intercepts of $q(x) = \dfrac{\left(x^2 + x - 6\right)(x - 4)}{x^2 - 4x}$

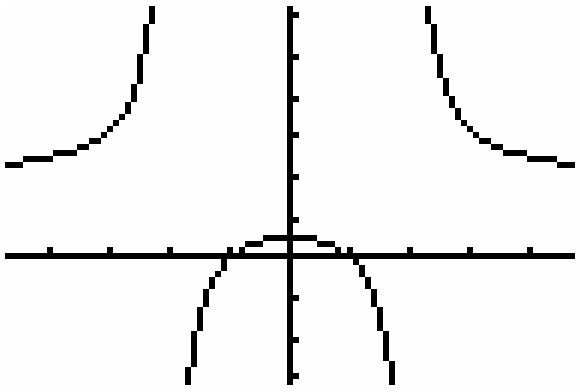
MEMORIZE THE FOLLOWING!!!!!!

There are three cases for possible horizontal asymptotes:

- If the degree of the numerator is greater than that of the denominator, there is no Horizontal Asymptote (like the previous two examples.)
- If the degree of the denominator is greater than the numerator, there is a Horizontal Asymptote at $y = 0$
  - $f(x) = \dfrac{x-1}{x^2}$



- If the degrees of the numerator and denominator are the same, there is a Horizontal Asymptote at $y =$ the quotient of the leading coefficients
  - $f(x) = \dfrac{2x^2-2}{x^2-4}$



Déjà RE-Vu

The high school band is planning a trip to play at a college bowl game. The trip will cost **$500** per band member plus a **$2000** deposit for the whole group.

a) Write a function to represent the average cost of the trip per band member.

b) Graph the function. Identify a Relevant domain and range, *x*- or *y*-intercepts, discontinuities, and horizontal asymptotes.

c) Find the total cost per person if 50 band members attend the bowl game.

References:

http://logos.simpleplants.com/Schools-Education/thumbs/Schools-Classroom-Activities-Marching_Band_2.gif