Déjà Vu, It's Algebra 2!

Lesson 24

Radical Expressions, Functions, & Equations

What is the square root of 25?

We can write this question using a radical: $\sqrt{25}$

When we ask this question, we really want to answer the following, *"What number, times itself, is equal to 25?"* The answer is 5, since $5 \cdot 5 = 5^2$

Is there another answer???????? What about -5?

Since $(-5)(-5) = (-5)^2 = 25$, $\sqrt{25} = \pm 5$

In general:

$$\sqrt[n]{a}$$

Root index → n

Radical → $\sqrt{\ }$

Radicand → a

Example:

$\sqrt[4]{81} =$ ± 3

$\sqrt[3]{-125} =$ -5

$\sqrt[6]{-729} =$ no solution

$-\sqrt{4^{-1}} =$ $-\frac{1}{2}$

Properties of Radicals:

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$$\sqrt{36} = \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = \pm(2 \cdot 3) = \pm 6$$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$$\sqrt[3]{\frac{81}{8}} = \frac{\sqrt[3]{81}}{\sqrt[3]{8}} = \frac{3}{2} = 1.5$$

Example:

Simplify the following expression: $\sqrt[4]{\frac{16x^8}{5}}$

$$\frac{\sqrt[4]{16} \cdot \sqrt[4]{x^8}}{\sqrt[4]{5}} = \frac{2x^2}{\sqrt[4]{5}}\left(\frac{\sqrt[4]{5}}{\sqrt[4]{5}}\right)\left(\frac{\sqrt[4]{5}}{\sqrt[4]{5}}\right)\left(\frac{\sqrt[4]{5}}{\sqrt[4]{5}}\right) = \frac{2x^2\sqrt[4]{5^3}}{5} = \frac{2x^2\sqrt[4]{125}}{5}$$

A rational exponent is an exponent that can be expressed in the form $\frac{m}{n}$, where m and n are integers. Every radical expression can be written equivalently with rational exponents.

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Example:

$\left(\sqrt{x}\right)^3 =$

$x^{3/2}$

$\sqrt[4]{16x^3} =$

$16^{1/4} \cdot x^{3/4} = 2x^{3/4}$

$(-125)^{2/3} =$

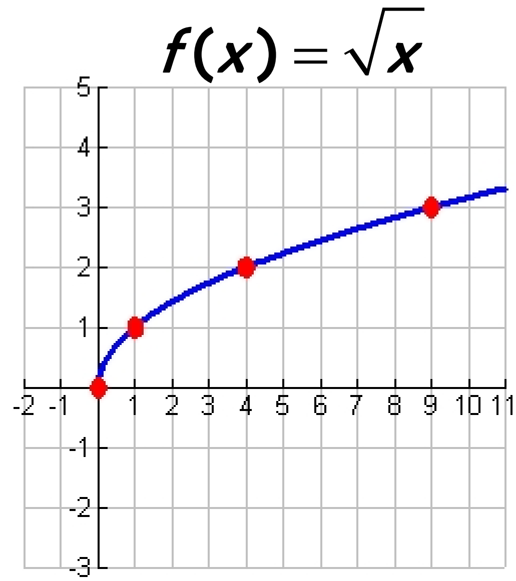
$\left(\sqrt[3]{-125}\right)^2 = (-5)^2 = 25$

A Radical Function is a function containing a radical. A square root function contains $\sqrt{x}$

What does the graph of $f(x) = \sqrt{x}$ look like??

(notice the indicated root is positive)

x	$f(x)=\sqrt{x}$	$(x, f(x))$
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$
16	$f(16)=\sqrt{16}=4$	$(16,4)$



We can graph transformations of this "parent" function of the form

$$g(x)=a\sqrt{b(x-c)}+d$$

Example:

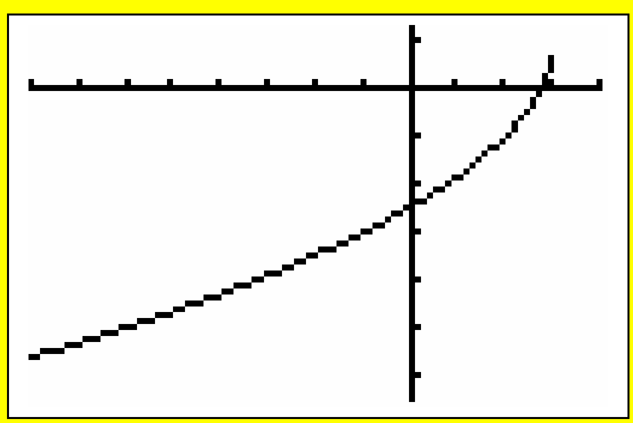
Sketch $g(x)=1-2\sqrt{3-x}$, the find the domain and range.

$g(x)=-2\sqrt{-(x-3)}+1$

x-axis reflection
vertical stretch by a factor of 2
y-axis reflection
right 3
up one

Domain: $\{x \mid x\le 3\}$

Range: $\{y \mid y\le 1\}$



Déjà RE-Vu

Solve:

$$\sqrt{x+18} = x-2$$

$\left(\sqrt{x+18}\right)^2 = (x-2)^2$

$x+18 = x^2-4x+4$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x=7$ or $x=-2$

check

$x=7$	$x=-2$
$\sqrt{7+18}\,"="\,7-2$	$\sqrt{-2+18}\,"="\,-2-2$
$\sqrt{25}\,"="\,5$	$\sqrt{16}\,"="\,-4$
$5=5$	$4\neq 4$

So only $x=7$ is a solution