Déjà Vu, It's Algebra 2!

Lesson 26

Piecewise Functions

The first person to fly at a speed greater than the speed of sound was Chuck Yeager. On October 14, 1947, flying in a X-1 rocket plane called the "Glamorous Glennis," at an altitude of 12.8 kilometers, Yeager was clocked at 299.5 meters per second. If Yeager had been flying at an altitude under 10.375 kilometers, his speed of 299.5 meters per second would not have "broken the sound barrier!"

This is because at different altitudes in earth's atmosphere, sound travels at different speeds!

It would be impossible to write one equation to model the speed of sound in the atmosphere as a function of altitude because of this.

. . . Enter Piecewise Functions!!!

A piecewise function is a function that is a combination of one or more functions. The rule, or equation, for a piecewise function is different for different parts, or pieces, of the domain.

Things like movie ticket prices as a function of age, a triathlete's performance as a function of time, and price to cross a bridge as a function of weight can all be modeled by piecewise functions.

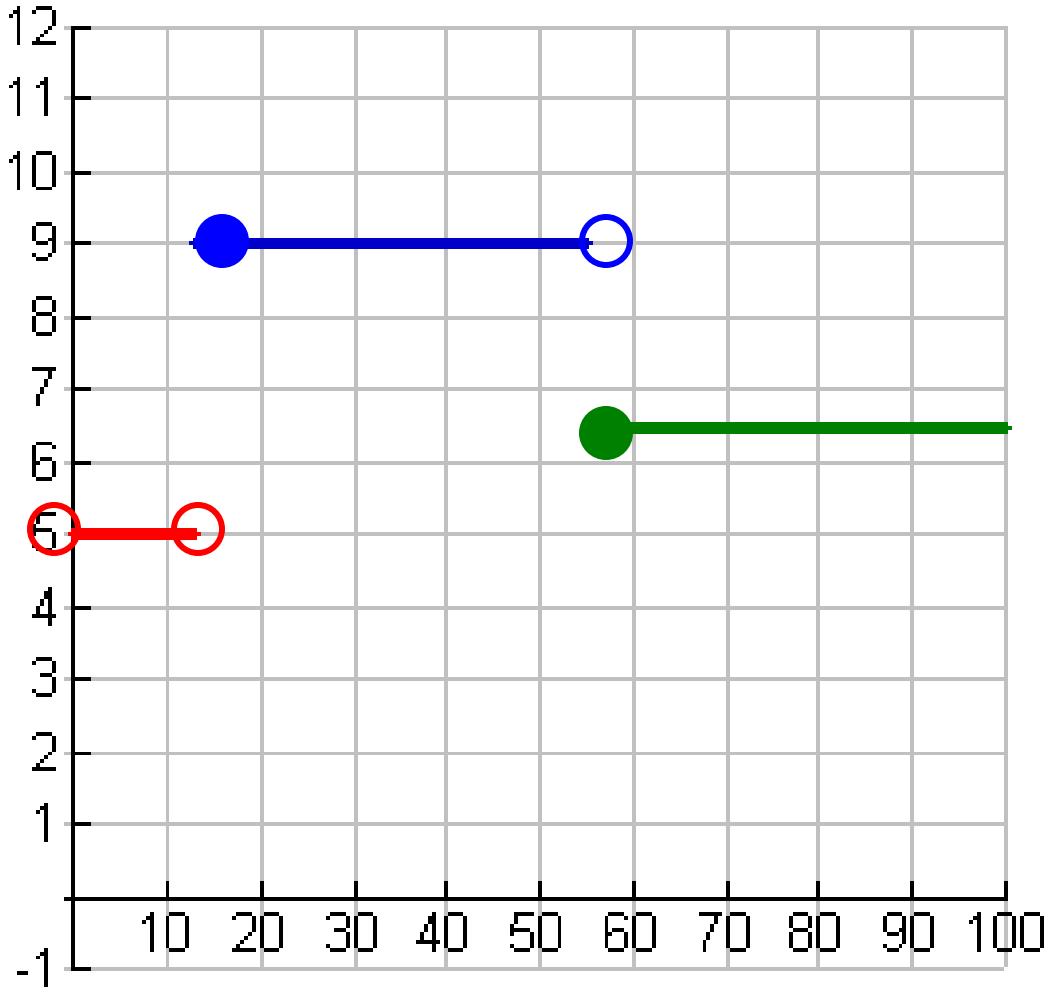
Example:

A movie theater charges the following rates for its prime-time tickets. Write a mathematical model for ticket price as a function of age, then sketch the graph.

Movie Tickets	
Age (yrs)	Price ($)
0-12	5.00
13-54	9.00
55+	6.50

$$P(x) = \begin{cases} 5, & 0 < x < 13 \\ 9, & 13 \le x < 55 \\ 6.5, & x \ge 55 \end{cases}$$

Movie Ticket Price Based on Age

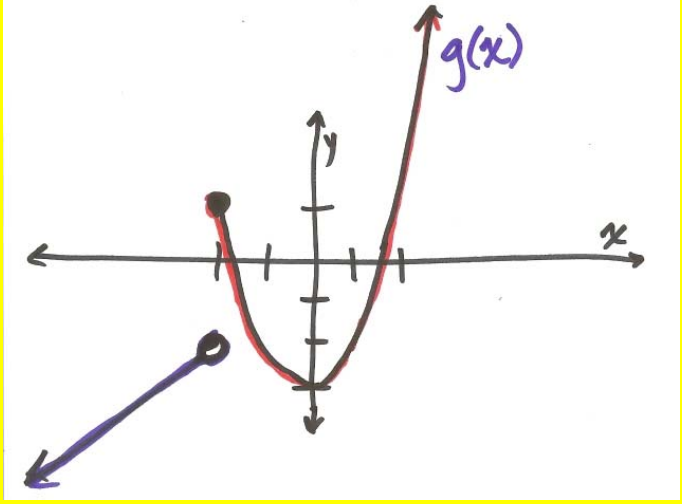


This piecewise, which is constant on each *x*-interval is a type of piecewise function called a step function.

But there are other types . . .

Example:

Sketch $g(x) = \begin{cases} 3x + 4, & x < -2 \\ x^2 - 3, & x \geq -2 \end{cases}$, then evaluate $g(-3)$, $g(-2)$, and $g(1)$.



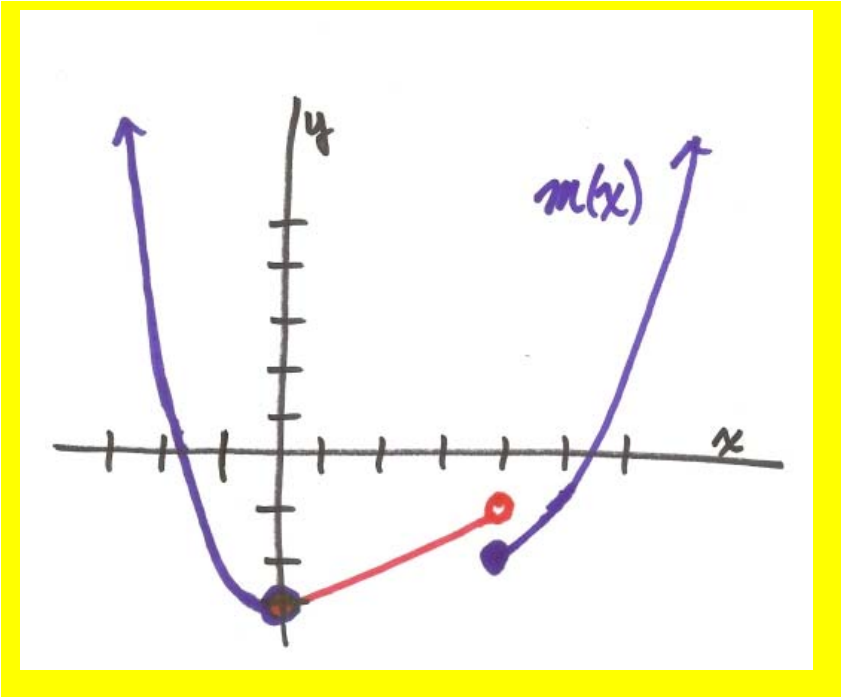
$g(-3) = -5$
$g(-2) = 1$
$g(1) = -2$

Notice the graph is still a function because at each transition point, the function is only defined at most once. This is very important when looking at the equation!!

Sometimes the graphs are made up of several pieces, connected at some, disconnected at others.

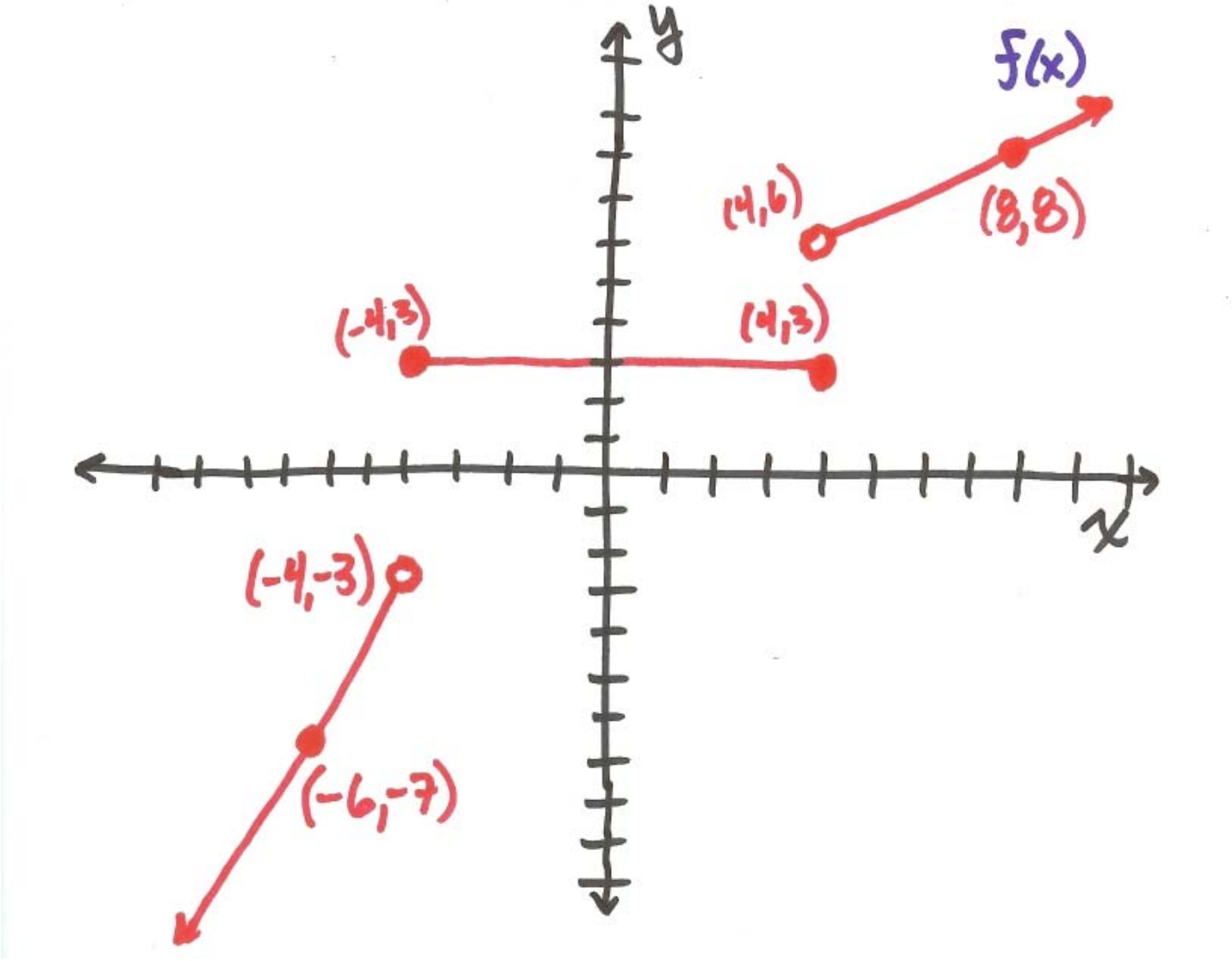
Example:

Sketch $m(x) = \begin{cases} x^2 - 3, & x < 0 \\ \frac{1}{2}x - 3, & 0 \le x < 4 \\ (x-4)^2 - 2, & x \ge 4 \end{cases}$



Example:

Write a piecewise function for the following graph.

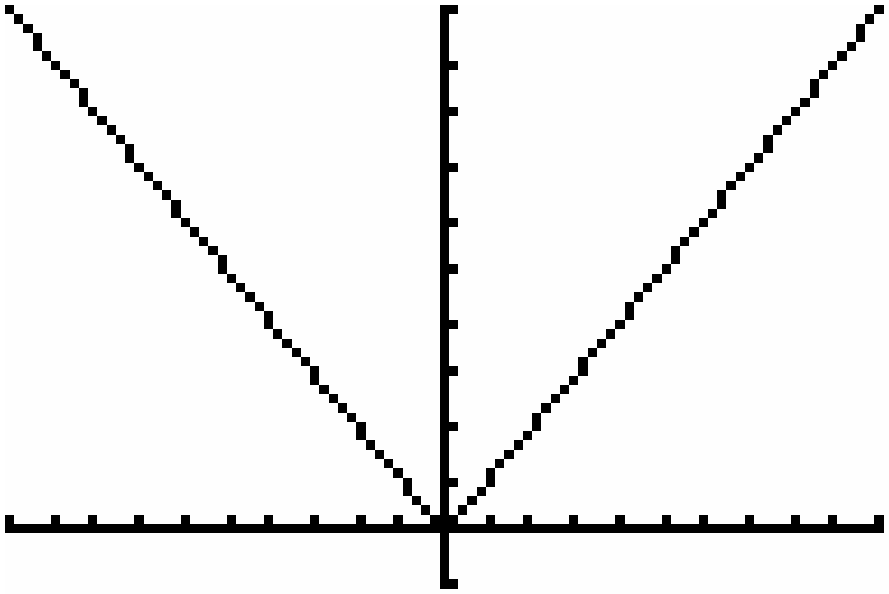


$$f(x)=\begin{cases}2x+5, & x<-4\\ 3, & -4\le x\le 4\\ \frac{1}{2}x+4, & x>4\end{cases}$$

Déjà RE-Vu

The absolute value function, which graphs all inputs as positive values, can be written as a piecewise function.

$f(x) = |x|$



$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

This is a helpful way to circumvent the absolute value symbol, anther of which is to call the function

$f(x) = \sqrt{x^2}$

Think of a piecewise function like the mythical Centaur, half man, half horse, changed at the torso.

Math is power!

References:

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