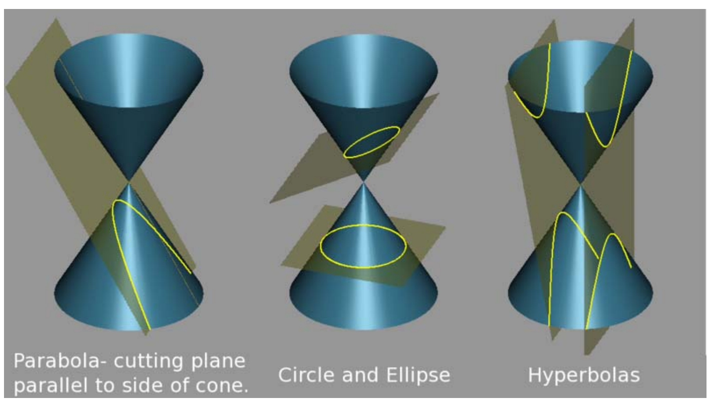
Déjà Vu, It's Algebra 2!

Lesson 28

Conic Sections Introduction & Circles

We have previously studied the parabola.

The parabola belongs to a family of curves called, Conic Sections, so called because they are formed from taking particular cross-sections of a double-knapped cone.



Conics, (as they are affectionately called) were studied by some the earliest Greek mathematicians, Apollonius and Hypatia for instance, and were valued because of their diverse practical applications.

Apollonius of Perga, one of the greatest Greek mathematicians of the time (circa 200 B.C.), appears to have been the first to have rigorously studied the conic sections. He applied his work to his study of planetary motion and used this to aid in the development of Greek astronomy.

There are four types of conic sections:

1. Circles,
2. Ellipses
3. Hyperbolas
4. Parabolas

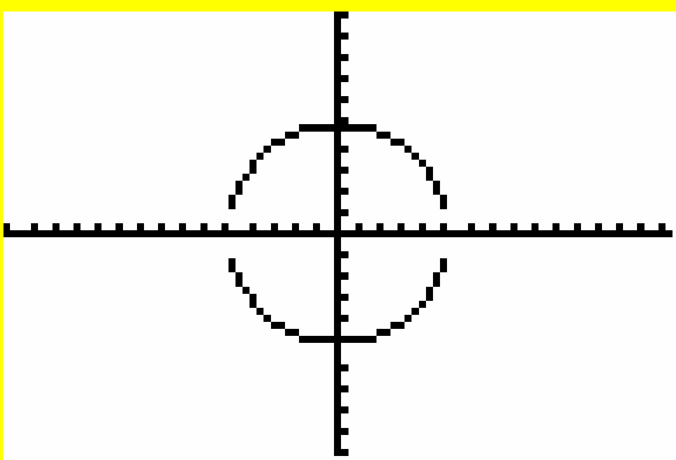
Although the parabolas we've seen in the past were functions, most conic sections are NOT functions!! This means they must be explicitly defined by two separate functions in order to graph them on your calculator.

Example:

Sketch the following conic section:

$$x^2 + y^2 = 25$$

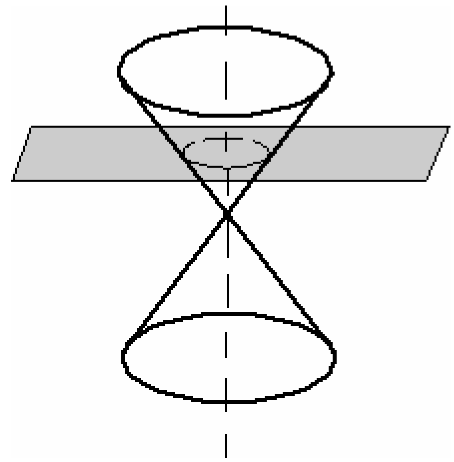
$y = \pm\sqrt{25 - x^2}$



This is a circle, centered at the origin, with a radius of 5 units

The circle is perhaps the most "famous" conic section.

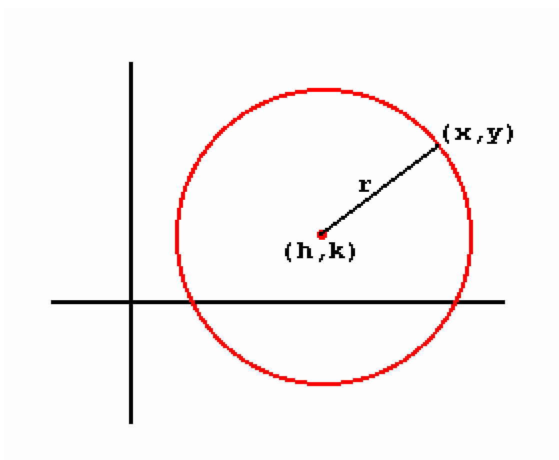
The circle is obtained by slicing a cone PARALLEL to the base of the cone. The circle has a very specific definition . . .



Definition: A circle is the set of all points that are a fixed distance, *r*, from a fixed point, *C*.

$r = radius$

$C = center$ at (h,k)



The standard form for the equation of a circle centered at (h,k) with a radius of r is

$$(x-h)^2+(y-k)^2=r^2$$

Example:

Write the equation of a circle centered at $(-2,5)$ with a diameter of 16.

$(x+2)^2+(y-5)^2=64$

Example:

Write the equation of a conic whose points are all equidistant from the point $(-4,11)$ that passes through the point $(5,-1)$.

$(5--4)^2+(-1-11)^2=r^2$

$81+144=r^2$

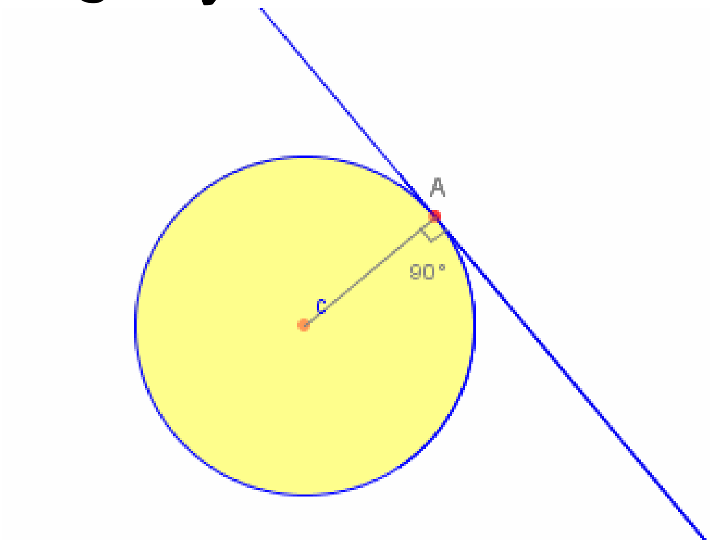
$r^2=225$

$r=15$

So the equation is

$(x+4)^2+(y-11)^2=225$

A tangent line is a line that intersects (touches) the circle at exactly one point. Recall from geometry that a tangent line is perpendicular to the radius at the point of tangency.



Example:

Write the equation of the line that is tangent to the circle $x^2 + y^2 = 29$ at the point $(2,5)$.

We need to find the slope of the line of the radius from the center to the point of tangency.

Center is at $(0,0)$. Tangent point is $(2,5)$

$$\text{slope} = m = \frac{5-0}{2-0} = \frac{5}{2}$$

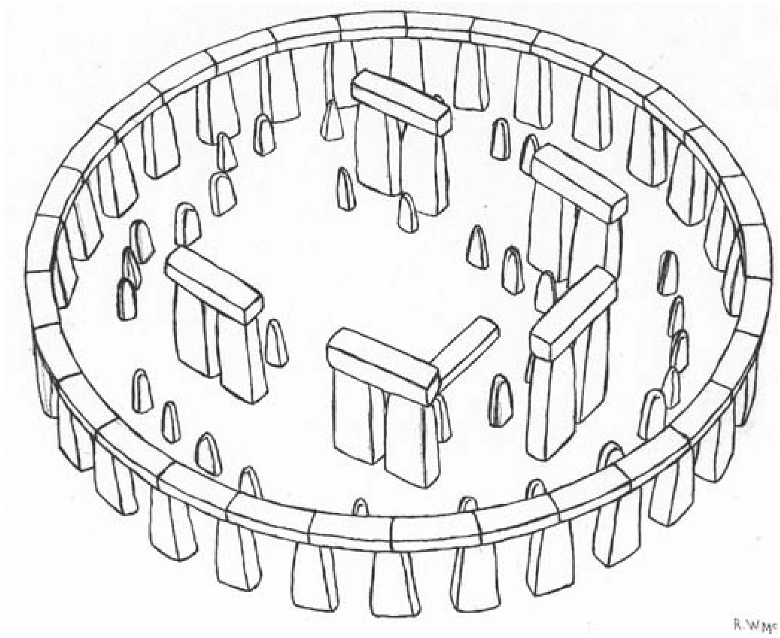
Recall that perpendicular slopes are "negative reciprocals" of each other, so the slope of the tangent line is $-\frac{2}{5}$. Now using the point-slope formula with slope $-\frac{2}{5}$ and point of tangency $(2,5)$

$$y-5 = \frac{2}{5}(x-2)$$

$$y = \frac{2}{5}x - \frac{4}{5} + 5$$

$$y = \frac{2}{5}x + \frac{21}{5}$$

Déjà RE-Vu



The outermost ring of the ancient monument Stonehenge can be modeled by the equation $x^2 + y^2 = 27{,}225$.

The Sarsen Circle, the center ring of stones usually associated with the monument, can be modeled by the equation $x^2 + y^2 = 2{,}916$.

a) The Heel Stone is located outside of the circles, approximately at the point $(0, 300)$. Find the maximum and minimum distances, in feet, to the Heel Stone from both the outer and inner circles. 465ft, 135ft; 354ft, 246ft

b) Two Station Stones surrounded by circular ditches are located within the outer circle. One stone is located at approximately $(-100, 100)$ and is surrounded by a ditch of radius 12 ft. Write an equation to model the ditch around this Station's Stone. $(x+100)^2 + (y-100)^2 = 144$

Math is everywhere!

References:

http://xahlee.org/SpecialPlaneCurves_dir/ConicSections_dir/conicSections.html
http://www.wikipedia.com
http://www.librarising.com/spirituality/images/hypatia.jpg
http://math.about.com/library/blconic.htm
http://www.doe.virginia.gov/Div/Winchester/jhhs/math/lessons/calc2006/day148.html
http://www.africa.upenn.edu/Hornet/LOCUST.GIF
http://www.mathopenref.com/tangent.html
http://www.seanet.com/~rwmcpherson/circle3b.jpg
http://www.cloudbait.com/archaeo/heelstone.jpg
http://witcombe.sbc.edu/stonehenge/stonehenge.html
http://picasaweb.google.com/dotemer/England/photo#5080461064925592946
http://upload.wikimedia.org/wikipedia/commons/8/8d/S7300095.JPG