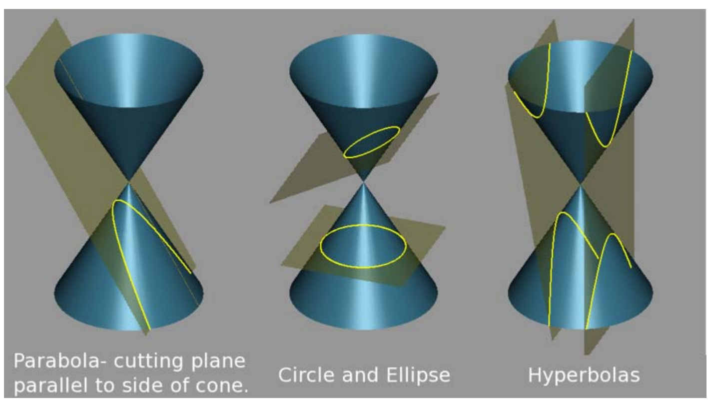
Déjà Vu, It's Algebra 2!

Lesson 28

Conic Sections Introduction & Circles

We have previously studied the parabola.

The parabola belongs to a family of curves called, Conic Sections, so called because they are formed from taking particular cross-sections of a double-knapped cone.



Conics, (as they are affectionately called) were studied by some the earliest Greek mathematicians, Apollonius and Hypatia for instance, and were valued because of their diverse practical applications.

There are four types of conic sections:

1. Circles,
2. Ellipses
3. Hyperbolas
4. Parabolas

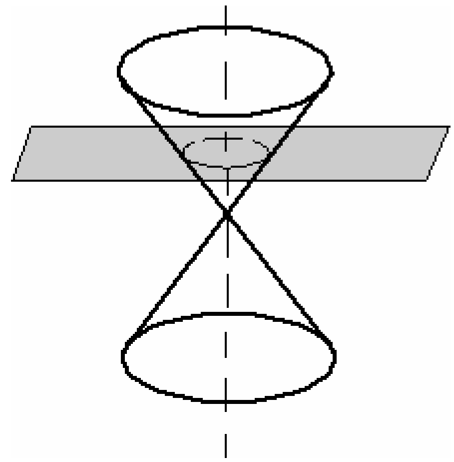
Although the parabolas we've seen in the past were functions, most conic sections are NOT functions!! This means they must be explicitly defined by two separate functions in order to graph them on your calculator.

Example:

Sketch the following conic section:

$$x^2 + y^2 = 25$$

The circle is perhaps the most "famous" conic section.

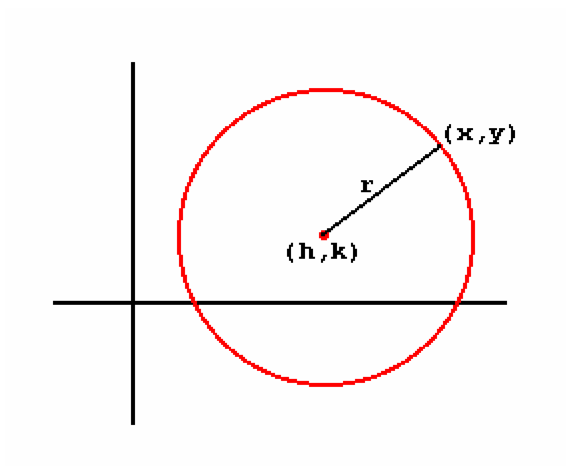


The circle is obtained by slicing a cone PARALLEL to the base of the cone. The circle has a very specific definition . . .

Definition: A circle is the set of all points that are a fixed distance, *r*, from a fixed point, *C*.

$r = radius$

$C = center$ at (h,k)



The standard form for the equation of a circle centered at (h,k) with a radius of r is

$$(x-h)^2+(y-k)^2=r^2$$

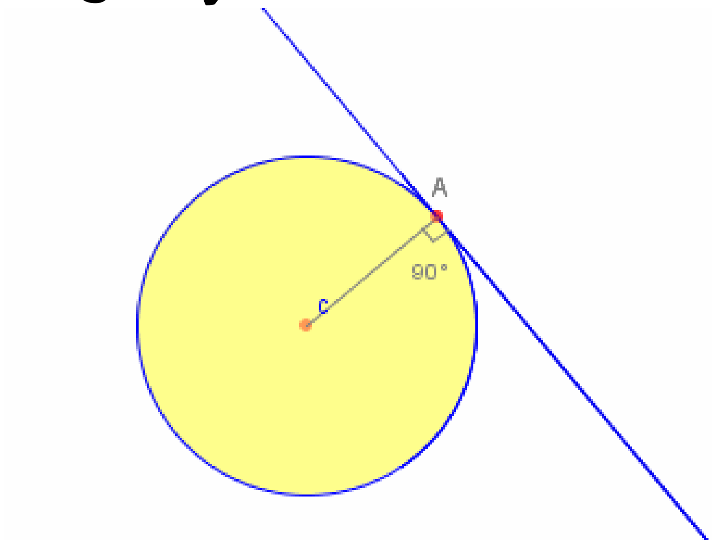
Example:

Write the equation of a circle centered at $(-2,5)$ with a diameter of 16.

Example:

Write the equation of a conic whose points are all equidistant from the point $(-4,11)$ that passes through the point $(5,-1)$.

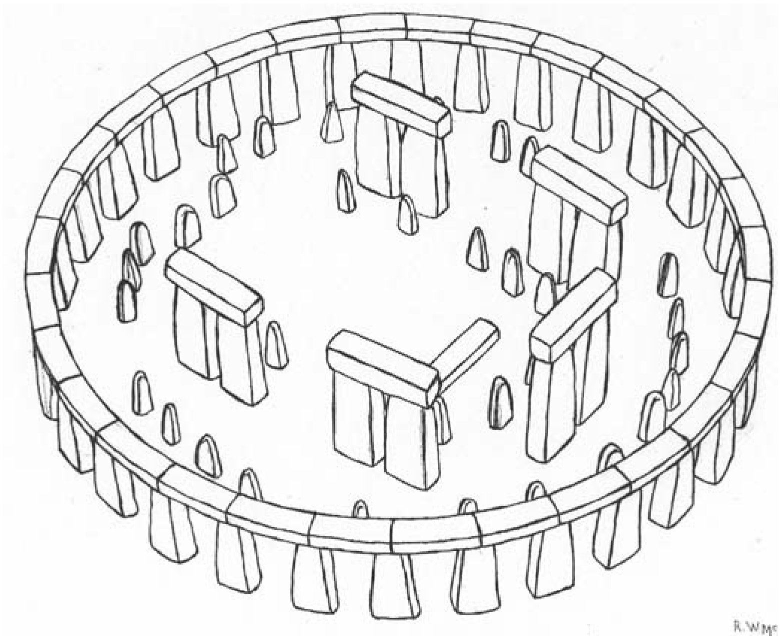
A tangent line is a line that intersects (touches) the circle at exactly one point. Recall from geometry that a tangent line is perpendicular to the radius at the point of tangency.



Example:

Write the equation of the line that is tangent to the circle $x^2 + y^2 = 29$ at the point $(2,5)$.

Déjà RE-Vu



The outermost ring of the ancient monument Stonehenge can be modeled by the equation $x^2 + y^2 = 27,225$.

The Sarsen Circle, the center ring of stones usually associated with the monument, can be modeled by the equation $x^2 + y^2 = 2,916$.

a) The Heel Stone is located outside of the circles, approximately at the point $(0,300)$. Find the maximum and minimum distances, in feet, to the Heel Stone from both the outer and inner circles.

b) Two Station Stones surrounded by circular ditches are located within the outer circle. One stone is located at approximately $(-100,100)$ and is surrounded by a ditch of radius 12 ft. Write an equation to model the ditch around this Station's Stone.

Math is everywhere!

References:

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