Déjà Vu, It's Algebra 2!

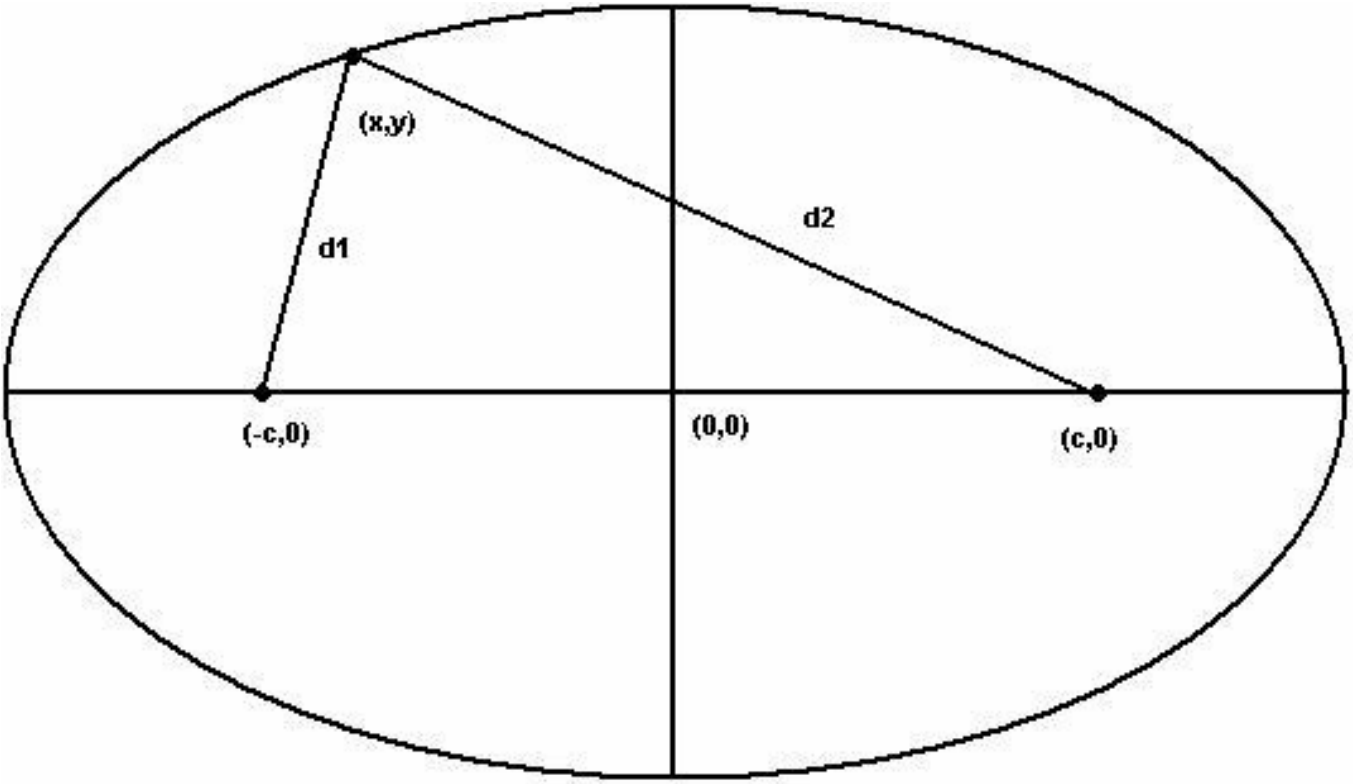
Lesson 29

Conic Sections continued: Ellipses

An **ELLIPSE** is formed by slicing a cone through laterally at an angle.

Locus Definition of an Ellipse:

The set of all points whose SUM of the distances to two fixed points, called the foci, is a constant.

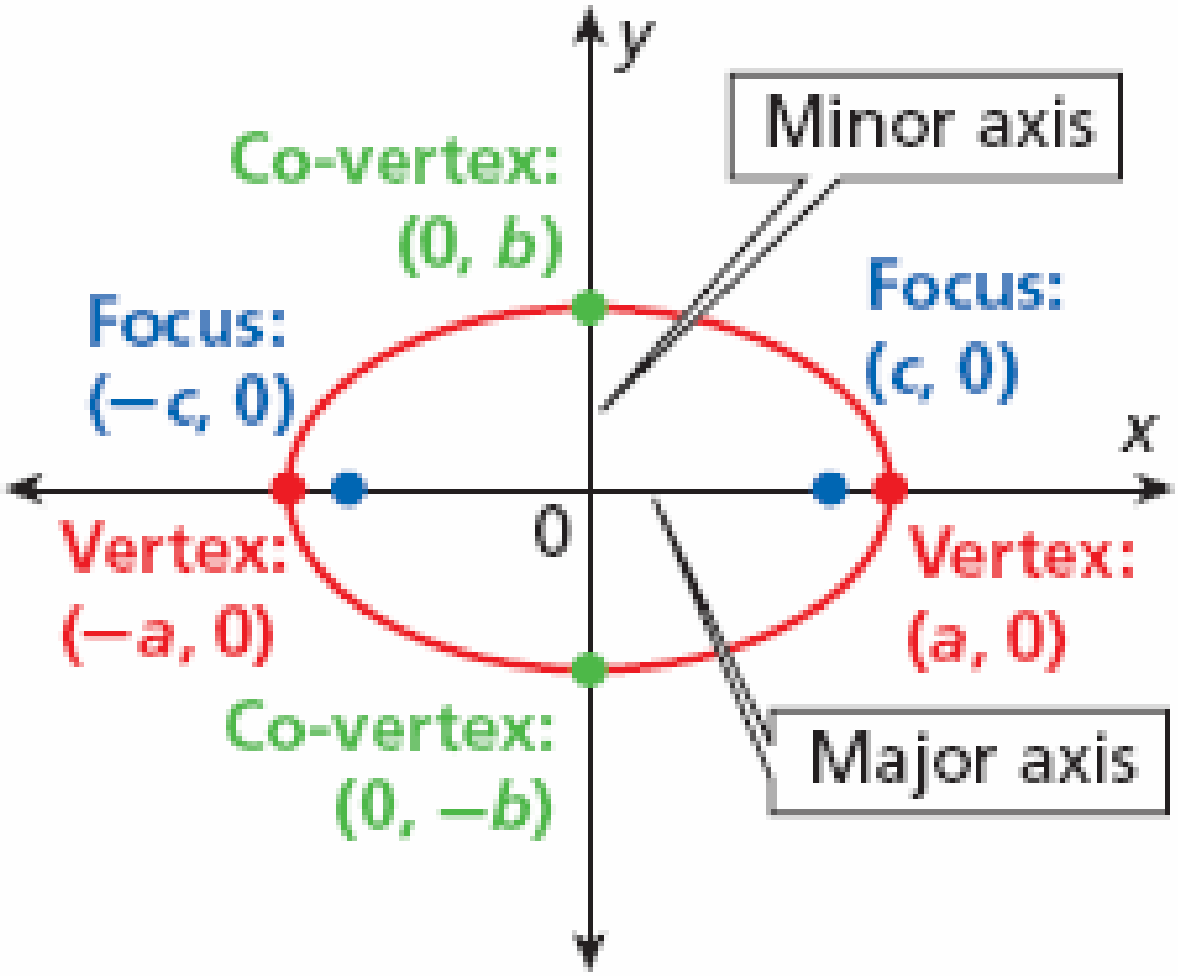


$$d_1 + d_2 = \text{ CONSTANT}$$

The ellipse can be thought of as a circle that has been dilated by different factors vertically and horizontally. In this fashion, an ellipse has TWO different radii: a longer one, called *a*, the semi-major axis, and a shorter one, called *b*, the semi-minor axis.

The endpoints of the major axis are called VERTICES. The endpoints of the minor axis are called CO-VERTICES.

The two foci are ALWAYS located on the longer, or major, axis. The distance from the center of an ellipse to either foci is called c or the FOCAL length.



The standard equation for an ellipse with center at $(0,0)$ is

Horizontal: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Vertical: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$$a > b$$

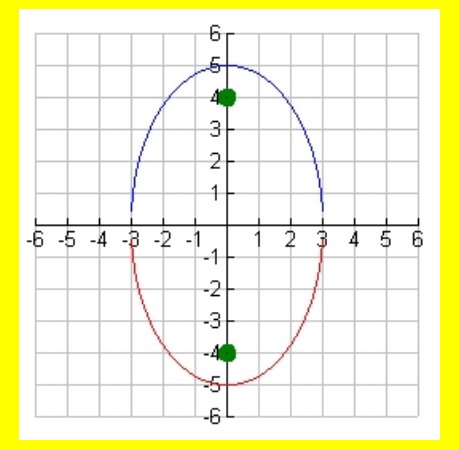
There is a very important relationship among the semi-major axis length , a, the semi-minor axis length, b, and the focal length, c.

$$c^2 = a^2 - b^2$$

Example:

Sketch the ellipse with the following equation.

$$25x^2 + 9y^2 = 225$$



Example:

Write an equation in standard form of an ellipse with a center at $(0,0)$, a vertex at $(-9,0)$, and a co-vertex at $(0,7)$.

$$\frac{x^2}{81} + \frac{y^2}{49} = 1$$

Example:

Write an equation in standard form of an ellipse with a center at $(0,0)$, a co-vertex at $(-7,0)$, and a focus at $\left(0,\sqrt{13}\right)$.

$$\frac{x^2}{62}+\frac{y^2}{49}=1$$

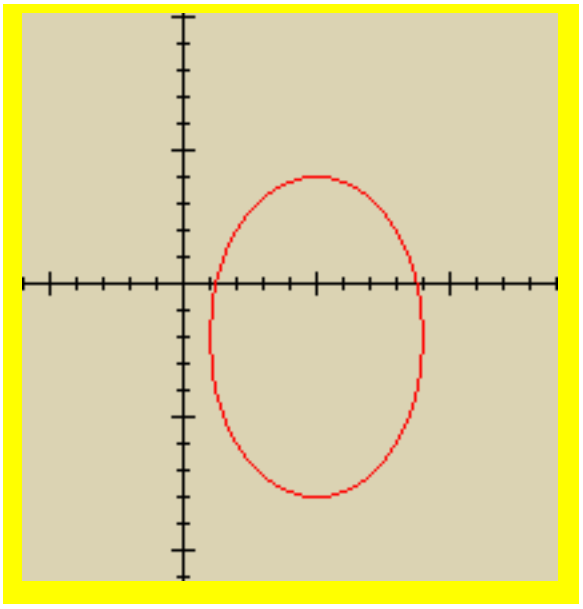
The standard equation of an ellipse with a center at (h,k) is given by the following equations:

Horizontal: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$

Vertical: $\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$

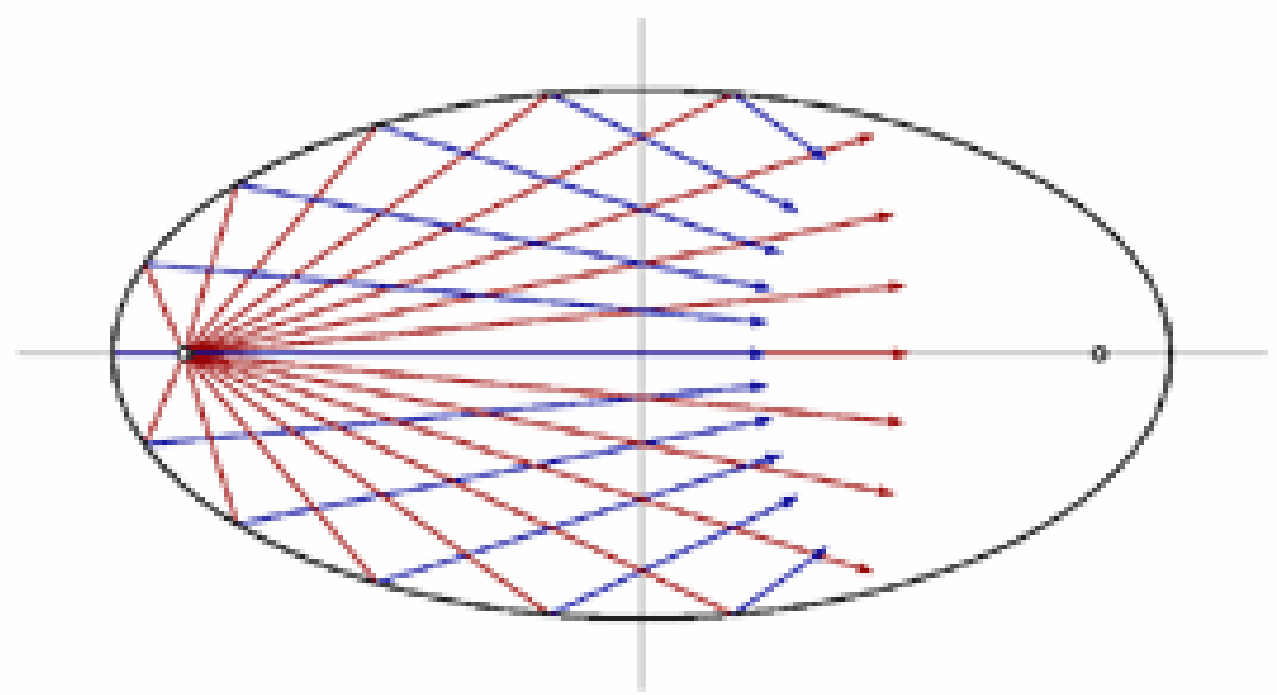
Example:

Graph the ellipse given by $\frac{(x-5)^2}{16}+\frac{(y+2)^2}{36}=1$



Déjà RE-Vu

As a result of their unique elliptical shapes, whispering galleries enable the smallest sound generated at one focus to be carried across the room to the other focus.



The whispering gallery at the Chicago Museum of Science and Industry is 47 ft. 4 in. long and 13 ft. 6 in. wide.

- Suppose that the center of the floor of the whispering gallery is located at the origin. Write an equation for the gallery floor.

$$\frac{x^2}{5041/9}+\frac{y^2}{729/16}=1$$

- Find the coordinates of the foci. How far are they apart?

$(\pm 22.68, 0)$; ≈ 45.36 *ft*

Math is everywhere!

References:

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