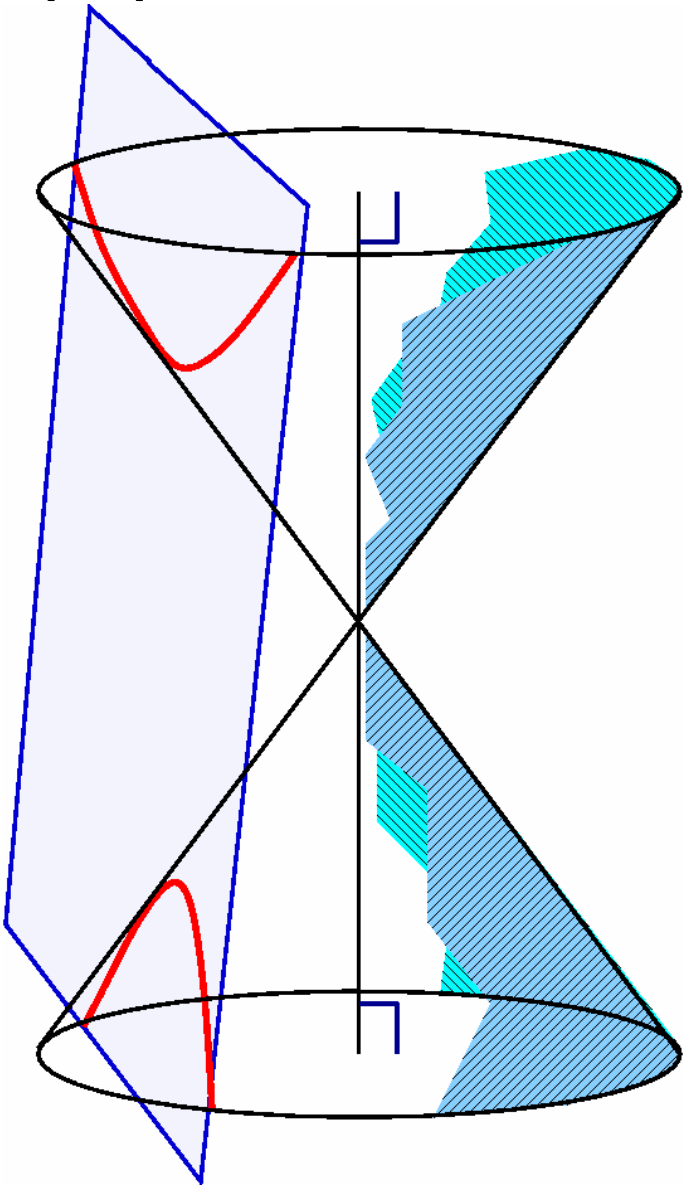
Déjà Vu, It's Algebra 2!

Lesson 30

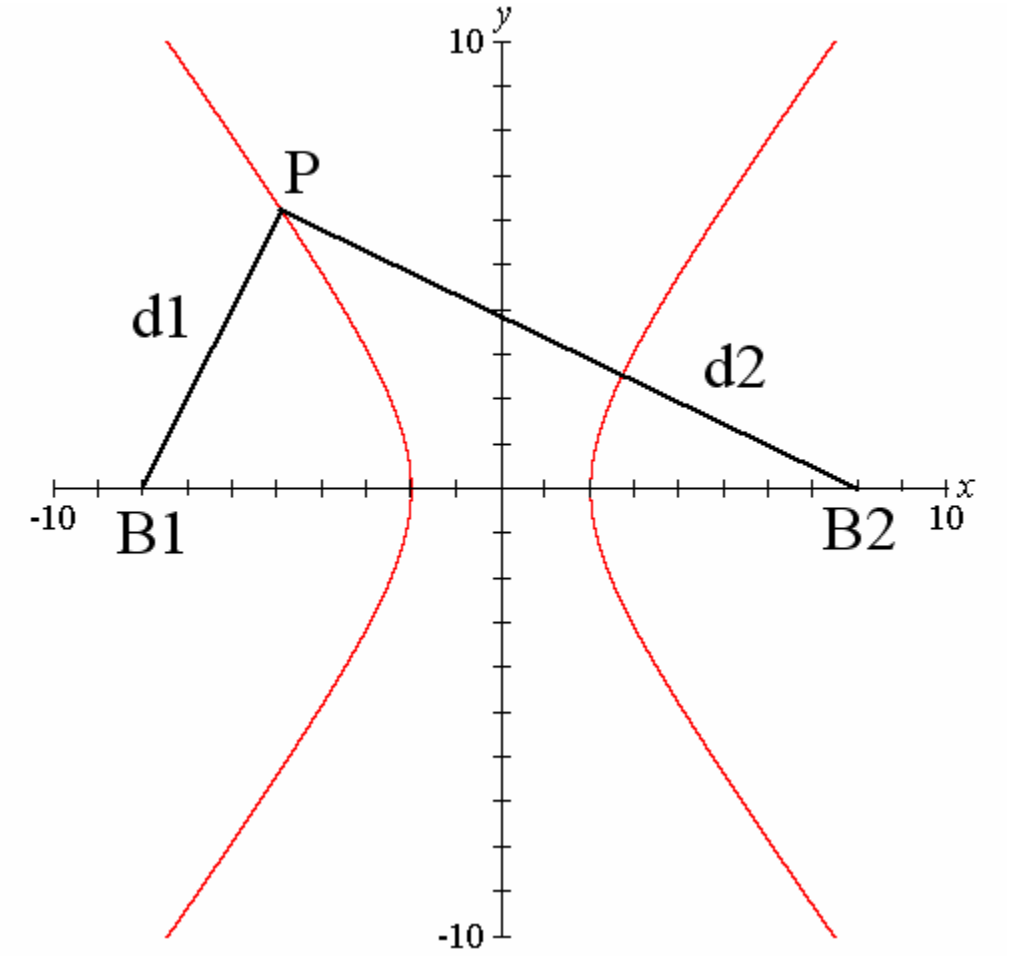
Conic Sections continued: Hyperbolas

A **HYPERBOLA** is formed by slicing a double-knapped cone perpendicular to the base.



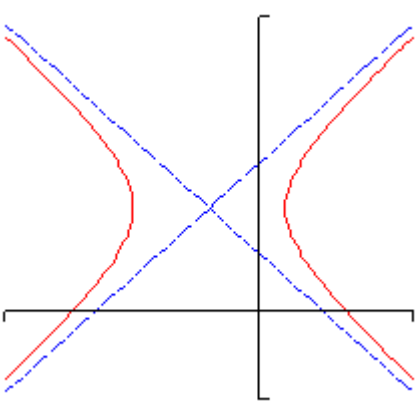
Locus Definition of a Hyperbola:

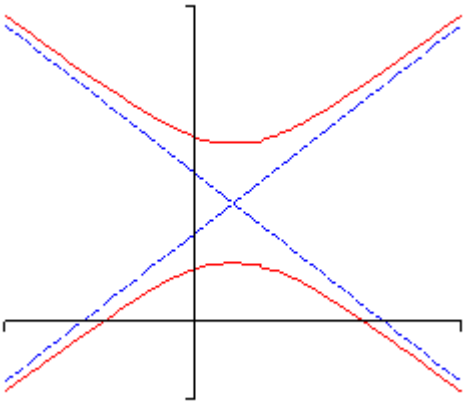
The set of all points whose DIFFERENCE of the distances to two fixed points, called the foci, is a constant.



$$|d_1 - d_2| = \text{CONSTANT}$$

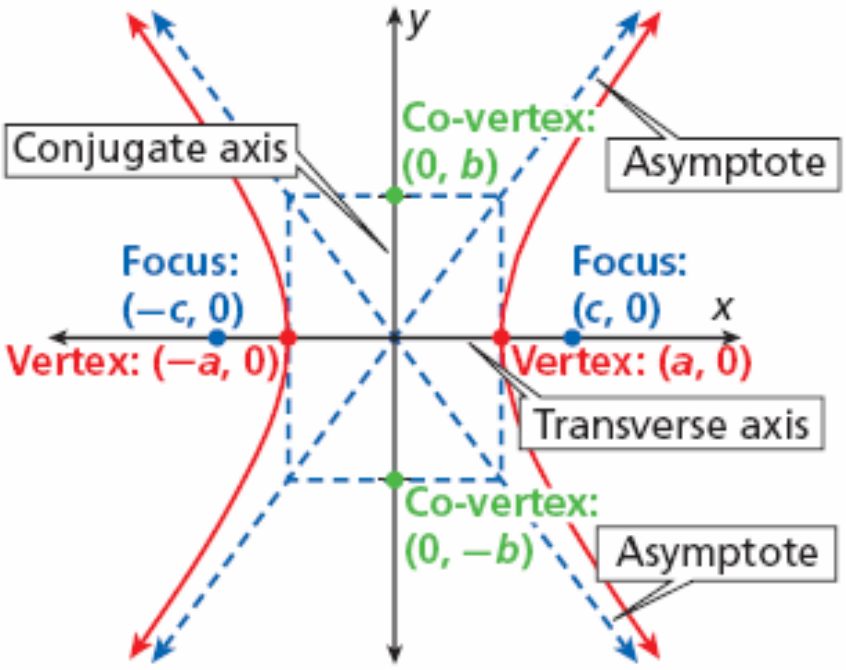
There are two basic varieties of hyperbola graphs:





There are two **standard forms** of the hyperbola, one for each type shown above. Here is a diagram of the horizontal variety as well as the information we can get from each one.

Hyperbola $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$



A hyperbola that opens vertically will be of the form:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The special relation among the variables *a*, *b*, and *c* is:

$$c^2 = a^2 + b^2$$

Let's try to graph one. . .

Example:

$$\frac{(x-3)^2}{25} - \frac{(y+1)}{49} = 1$$

Example:

Graph $\frac{y^2}{9} - (x+2)^2 = 1$

Aside from being able to graph hyperbolas given the equation, it is important to be able to write equations from a graph or given information.

Example:

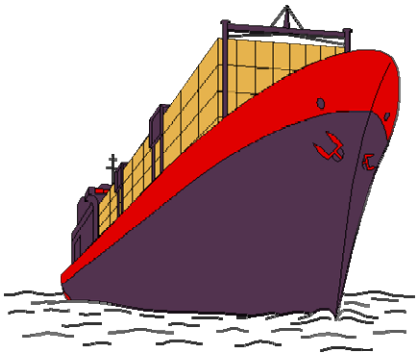
Find the equation of a hyperbola with center (1, 1), vertex (3, 1) and focus at (5, 1).

Déjà RE-Vu

Reflective Property of a Hyperbola:
Like other conics, a hyperbola can be made into an excellent reflector of sound, light, and other waves.

Long-Range Navigation:
Hyperbolas and radio signals are the basis of the LORAN (long-range navigation) system.

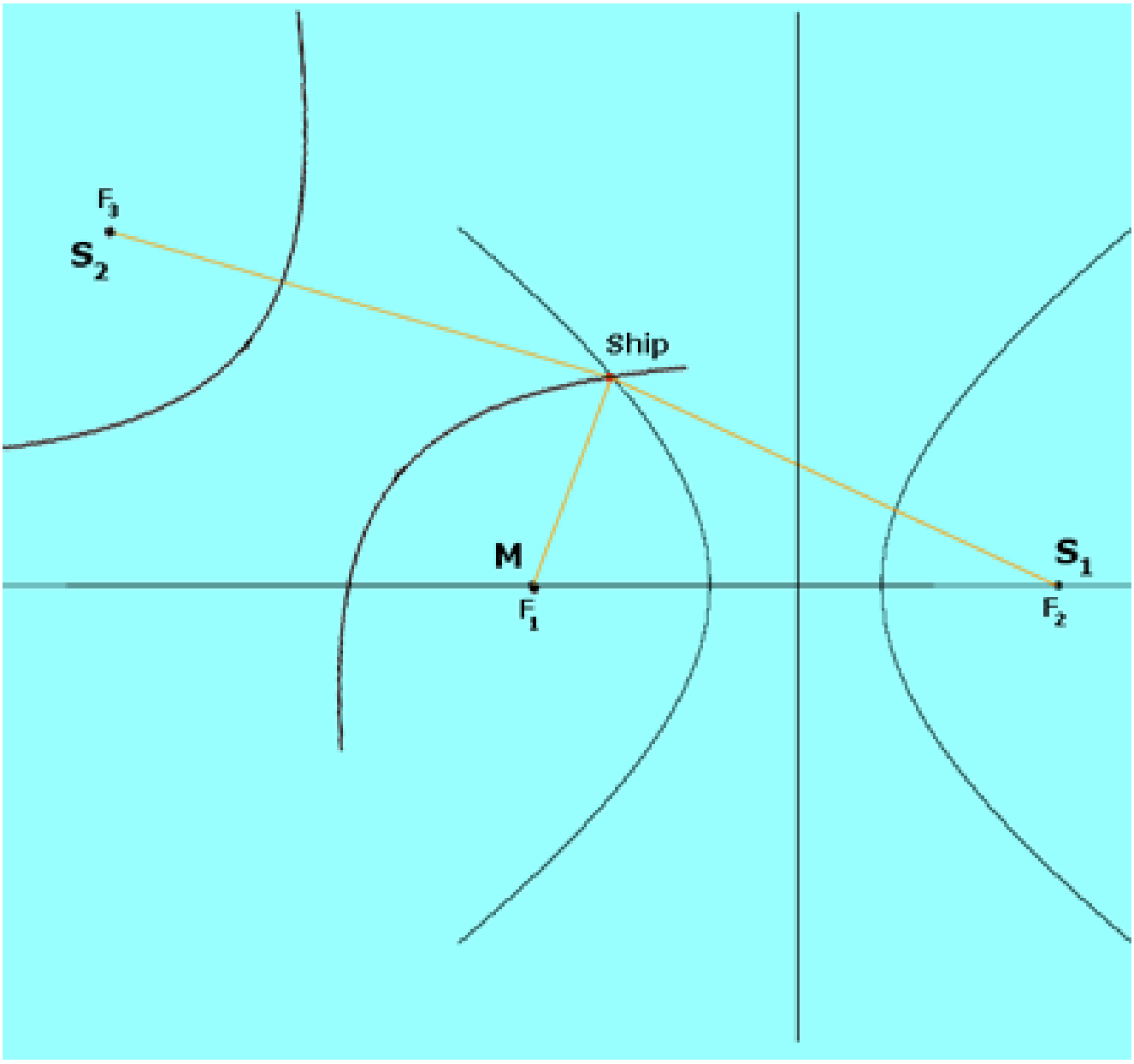
Developed in the US in 1940, LORAN works by comparing time differences between radio transmitters. Radio transmitters are setup in a chain of three or more are separated by hundreds of miles. A chain will have one master transmitter and a series of secondary transmitters. The stations constantly transmit signals with precise timing information.



A LORAN receiver compares the difference between the timing signals from the master-secondary transmitters and measures the difference. With this information, your position somewhere on a curved line from the transmitter can be determined. To find out where you are on the curved line, a second signal from another chain is

required. The second chain gives another curved line indicating your position on a curved line.

Where the curved lines intersect from each of the chains is your exact position.



Math is everywhere!

References:

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