Déjà Vu, It's Algebra 2!

Lesson 32

Permutations & Combinations

Permutations

Let's say we want to arrange the three letters of the word **DOG** into two-letter groups where **OG** is different from **GO** and no letters are repeated.

The easiest way to figure this out is to simply list them as follows:

In this case, since order matters, we are finding the number of *permutations* of size 2 that can be taken from a set of size 3.

We often write this as either ${}_nP_r$ or $P(n,r)$

In our case of the word **DOG**, we'd write it as ${}_3P_2$ or $P(3,2)$

Listing and counting the possibilities is a great method . . . if the list is small, but what if we want to find all the 4-letter permutations (without repeat) of the word **CUNEIFORMS**, a 10-letter word?!?

What is ${}_{10}P_4$?

Rather than list them, we can use the Fundamental Counting Principal.

There are 10 possibilities for the first letter, 9 for the second, 8 for the third, and 7 for the last letter. We can find the total number of 4-letter permutations by multiplying $10 \times 9 \times 8 \times 7 = 5040$.

The pattern above is *part* of what's called a factorial.

The factorial function (symbol: !) just means to multiply a series of descending natural numbers.

Examples:

4! = 4 × 3 × 2 × 1 = 24

7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040

1! = 1

Note: it is generally agreed that 0! = 1. It may seem funny that multiplying no numbers together gets you 1, but it helps simplify a lot of equations.

To obtain only the part 10 x 9 x 8 x 7, we need to divide 10! (because there are 10 letters total), by $(10-4)! = 6!$ (subtracting from the total number of letters the number of objects we are choosing in each permutation.) It looks like this . . .

$$ {}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1} $$

$$ = 10\times9\times8\times7 = 5040 $$

From this example, we can obtain the general formula for finding the number of permutations or size *r*, without repetition, taken from *n* objects.

$$ {}_nP_r = P(n,r) = \frac{n!}{(n-r)!} $$

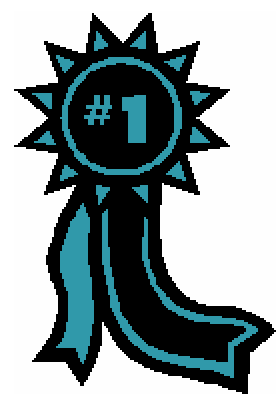
(NO REPEATS, ORDER MATTERS)

For our **DOG** example, this was

$$ {}_3P_2 = \frac{3!}{(3-2)!} = \frac{6}{1} = 6 $$

Example:

How many different ways can first, second, and third places be awarded to a group of 15 contenders?

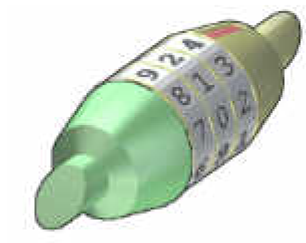


Note: There is a factorial button on your calculator, found under "MATH, PRB, 4."

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

Example:

If you were trying to crack a combination lock with 4 sliders, each numbered 0-9, how many attempts would you have to try before you were guaranteed to open it?



COMBINATIONS

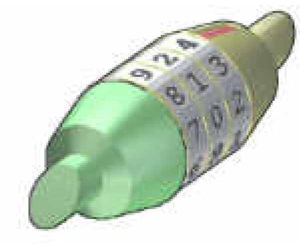
In English, we use the word "combination" loosely without thinking if the order of things is important. For instance:

- *"My diet consists of a combination of fruits, vegetables, dairy, meat, and math."*

- *"The combination to my safe is 1496"*

In Mathematics, we have a much more precise and formal language.

- If the order does NOT matter, it is a Combination.
- If the order DOES matter, it is a Permutation.



So we really should call this a Permutation Lock!

Think of a Permutation as an ordered Combination, where Position matters.

Let's return to the DOG example.
If we wanted to find the number of Combinations of size 2 without repeated letters that can be made from the 3 letters in DOG, order doesn't matter. OG is the same as GO. We can list the combinations:

DO DG OG

We can say "3 choose 2" and write it in any one of the three ways:

$$ {}_3C_2 = C(3,2) = \binom{3}{2} $$

What if we want to find the number of 4-letter combinations of the aforementioned 10-letter word **CUNEIFORMS**? Again, we don't want to have to write all the combos out.

Since we already know that ${}_{10}P_4 = 1050$, we can use this information to find ${}_{10}C_4$. The number of combinations should be much smaller than the number of permutations. This is because while arrangements of **CUNE, CUEN, CNUE, NUEC**, ... are unique permutations, they are the SAME combination!! We must then "reduce" our number of 5040 by the number of permutations of these 4-letter arrangements, $4 \times 3 \times 2 \times 1 = 4!$
The formula then becomes . . .

$$ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!\,r!} $$

For our **CUNEIFORMS** example,

To verify our **DOG** example,

Example:

A lottery game pays the jackpot if an individual correctly chooses the 6 winning numbers from a field of 50. What are his chances of winning?

Déjà RE-Vu

How many unique permutations of the word MISSISSIPPI are there?

Math is everywhere!

References:
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