Chapter 9.4: Conic Sections—Parabolas

The last conic section we will study is one with which your are already familiar: parabolas. Since Algebra I (and all through Algebra II), you have worked with parabolas, not even knowing they were a conic section. We will soon see that parabolas may open vertically, which we know well, and also horizontally.

To visualize geometrically how parabolas come from a cone, imagine slicing a single right circular cone with a plane that cuts through the base at an angle and comes out the other side, as shown in the figure at right.

Algebraically, we have a bit different type of general equation; instead of having two squared variables, we have only **ONE SQUARED VARIABLE**!

General Equation of a Parabola: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

The General Equation of a Parabola is given as

Parabola: $Ax^2 + Dx + Ey + F = 0$ or $Cy^2 + Dx + Ey + F = 0$

The parabola will open according to the **NON-SQUARED VARIABLE**!

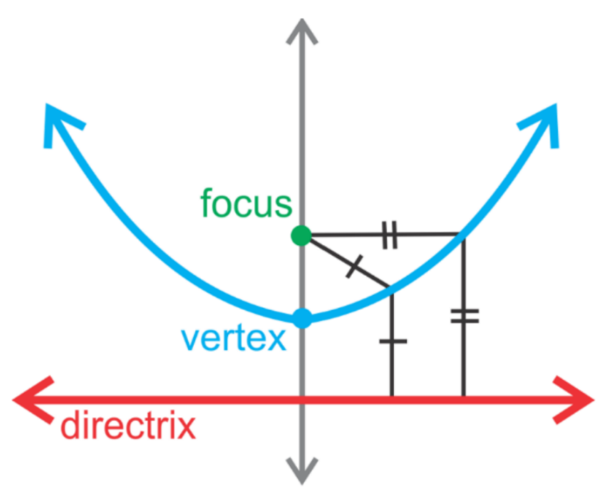
Locus Definition of a Parabola

A Parabola is the set of all coplanar points equidistant from a given point (the **focus**) and a given line (the **directrix**).

A parabola has no center like the other conics, so we define the vertex to be at (h, k). The focal length, the distance from the vertex to the focus, is c. The happens to also be the distance from the vertex to the directrix.

In picture at right, if the vertex is at the origin, the focus is at the point $(0, c)$, and the equation of the directrix, a horizontal line, is $y = -c$.

*In Algebra II and in other courses, the letter for focal length is sometimes called p.
**While all other conics have a directrix too, we will only study the directrix for the parabola.

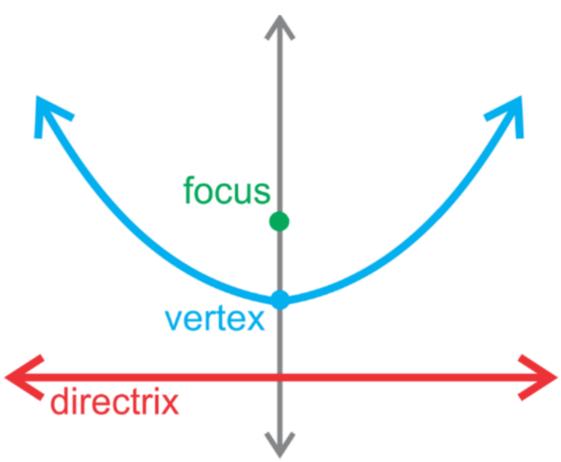


Example 1:
Given the general equation $2x^2 - 4x + y + 4 = 0$, put the equation into standard form. Sketch the parabola and identify the vertex, focus, directrix, domain, range, axis of symmetry and eccentricity.

In general, if the parabola has a vertex at (h, k), we can define the other features in terms of h and k. Here's what the standard equations for parabolas looks like for both the vertical and horizontal:

Vertical: $(x - h)^2 = 4c(y - k)$
(y NOT squared)

$\boldsymbol{x^2}$



$c > 0$ (opens up)

**If $c < 0$, opens down

Vertex: (h, k)

Focus: $(h, k + c)$

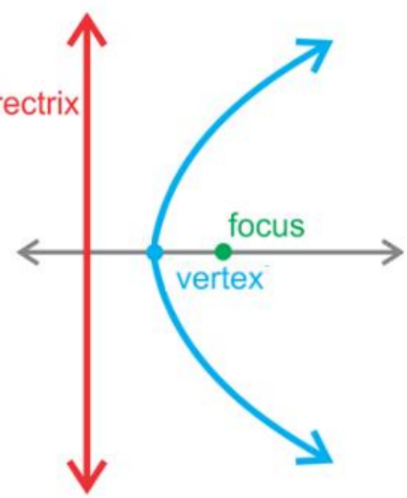
Directrix: $y = k - c$

Eccentricity: $e = 1$

Axis of Symmetry: $x = h$

Horizontal: $(y - k)^2 = 4c(x - h)$
(x NOT squared)

$\boldsymbol{y^2}$



$c > 0$ (opens right)

**If $c < 0$, opens left

Vertex: (h, k)

Focus: $(h + c, k)$

Directrix: $x = h - c$

Eccentricity: $e = 1$

Axis of Symmetry: $y = k$

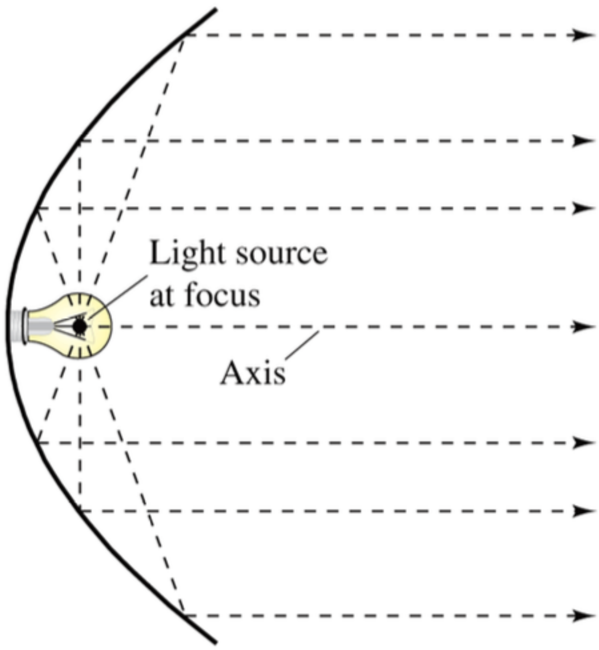
Example 2:
Given $x^2 = 12y$, sketch the parabola and identify the vertex, focus, directrix, domain, range, axis of symmetry and eccentricity.

Example 3:
Given $\frac{1}{8}(y-3)^2 = 2 - x$, sketch the parabola and identify the vertex, focus, directrix, domain, range, axis of symmetry and eccentricity.

Example 4:
Calculator Graph the parabola $(y-3)^2 = -8(x-2)$ from the previous example on the calculator.

Example 5:
Determine the equation of the parabola with a focus at $(3, -4)$ and a directrix of $y = 2$.

A parabola has a very useful reflective property:



Anything that emanates from the focus will reflect off the parabola into parallel lines, like in the image at left. The parabola (and paraboloid) make good receivers as well, in which case the arrows are reversed!

Example 6:
A parabolic communications antenna has a focus 6 ft from the vertex of the antenna. Find the width of the antenna at the vertex. This is known as the **latus rectum** of the parabola.