

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 2.2—Limits & Continuity**Give simplified, exact values for all answers. **No Calculator is Permitted.****I. Multiple Choice**\_\_\_\_\_ 1. Suppose  $\lim_{x \rightarrow 5} f(x) = 4$ . Which of these statements must be true?I. The range of  $f$  contains 4.II.  $f(5) = 4$ III. As  $x$  approaches 5 from the left,  $f(x)$  approaches 4.

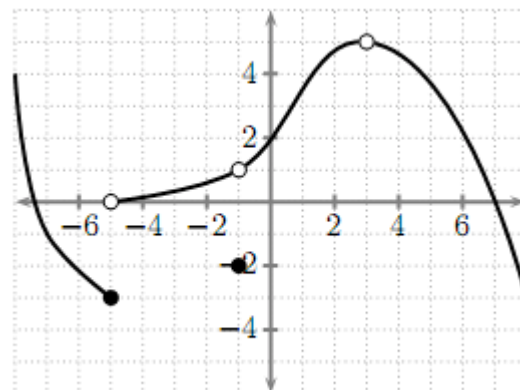
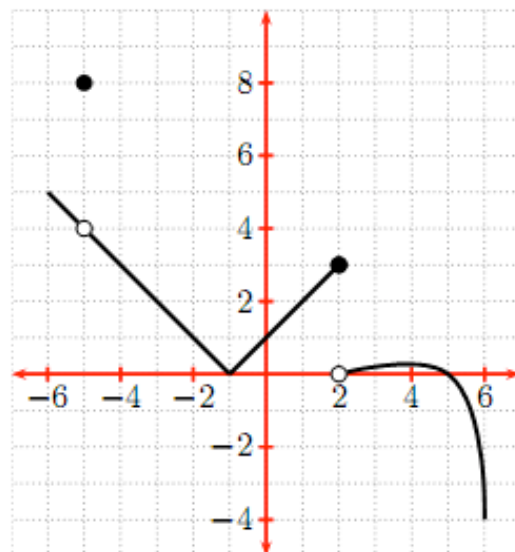
(A) II only (B) III only (C) I and III only (D) I, II, and III only (E) None of them

Use the graph of  $f(x)$  at right to answer questions 2 through 6.\_\_\_\_\_ 2. Determine  $\lim_{x \rightarrow 2^-} f(x)$ .(A) 0 (B) 3 (C) -4 (D)  $\frac{3}{2}$  (E) DNE\_\_\_\_\_ 3. Determine  $\lim_{x \rightarrow 2^+} f(x)$ .(A) 0 (B) 3 (C) -4 (D)  $\frac{3}{2}$  (E) DNE\_\_\_\_\_ 4. Determine  $\lim_{x \rightarrow 2} f(x)$ .(A) 0 (B) 3 (C) -4 (D)  $\frac{3}{2}$  (E) DNE\_\_\_\_\_ 5. Determine  $\lim_{x \rightarrow -5} f(x)$ .

(A) 4 (B) 8 (C) 0 (D) 6 (E) DNE

\_\_\_\_\_ 6. Determine  $f(-5)$ .

(A) 4 (B) 8 (C) 0 (D) 6 (E) DNE

\_\_\_\_\_ 7. The graph of a function  $f(x)$  is shown at right. Use the graph to determine all the values of  $x$  at which  $f$  fails to be continuous on the open  $x$ -interval  $x \in (-8, 8)$ .(A) -5, 3 (B) -1, 3 (C) -5, -1  
(D) -5, -1, 3 (E)  $f$  is continuous everywhere

\_\_\_\_\_ 8. Evaluate  $\lim_{x \rightarrow \infty} \frac{-4x^2 + 3x^5 + 1}{-7x^5 + x^2 + 2}$

(A) 0      (B)  $-\frac{3}{7}$       (C)  $\frac{4}{7}$       (D)  $\infty$       (E)  $-\infty$

\_\_\_\_\_ 9. Evaluate  $\lim_{x \rightarrow -\infty} \frac{-5x^7 - 8x^5 + 2}{-3x^6 - x^3 + x^2}$

(A) 0      (B)  $\frac{5}{3}$       (C)  $-\frac{5}{3}$       (D)  $\infty$       (E)  $-\infty$

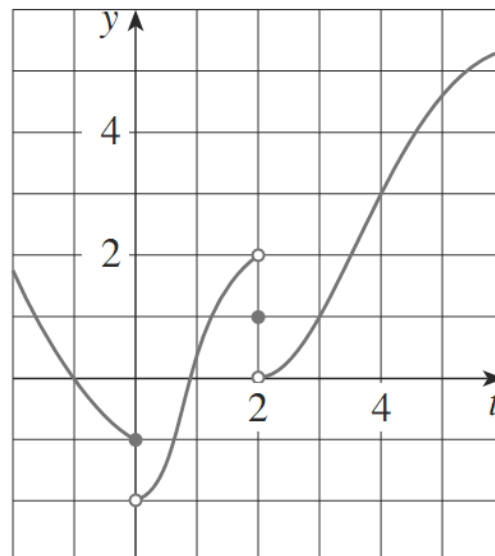
\_\_\_\_\_ 10. Evaluate  $\lim_{x \rightarrow -5^+} \frac{-2}{x+5}$  (you might want to sketch a graph first.)

(A) 0      (B) -2      (C)  $-\frac{1}{5}$       (D)  $\infty$       (E)  $-\infty$

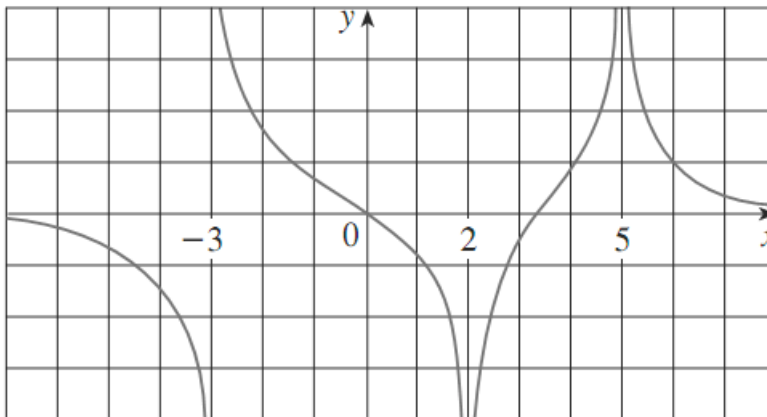
## II. Short Answer

11. Given the graph of  $f(x)$  at right, for each of the following, use the 3-step definition of continuity at a point to determine if the function is continuous at the indicated point. Be sure to show the analysis of all 3 steps, with correct notation, and a complete sentence at the end stating your conclusion with justification.

- (a) at  $x = 0$                       (b) at  $x = 2$                       (c) at  $x = 4$



12. For the function  $R$  whose graph is shown below, state the following:



(a)  $\lim_{x \rightarrow 2} R(x)$

(b)  $\lim_{x \rightarrow 5} R(x)$

(c)  $\lim_{x \rightarrow -3^-} R(x)$

(d)  $\lim_{x \rightarrow -3^+} R(x)$

(e)  $\lim_{x \rightarrow -3} R(x)$

13. For each function,  $f(x)$ , determine (i)  $\lim_{x \rightarrow -\infty} f(x)$  (ii)  $\lim_{x \rightarrow \infty} f(x)$  (iii) any equations of HA's

(a)  $f(x) = \frac{-5x+1}{x+7x^2+4}$

(b)  $f(x) = \frac{3x^6-2x^3+\pi}{8-9x^4}$

(c)  $f(x) = 6x^2 - 100x - 5000$

(d)  $f(x) = 3 - 2x^3$

(e)  $f(x) = \frac{12x+17-53x^2}{x^2+22x-114}$

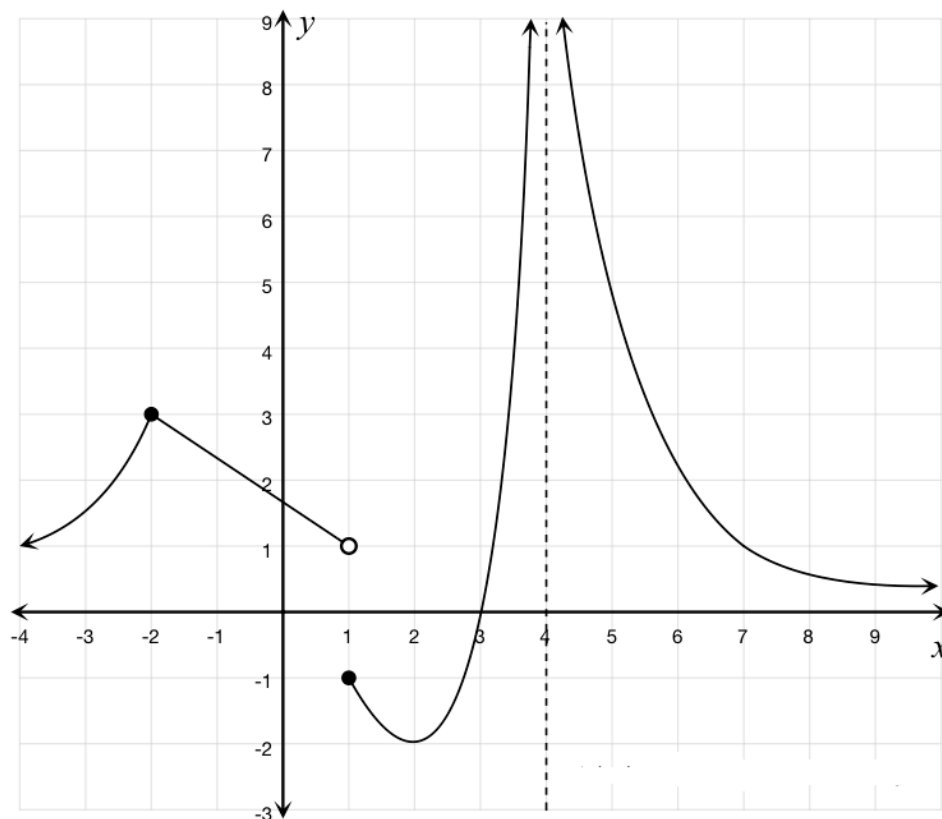
(f)  $f(x) = -4\sqrt{x+7}$

15. Determine if the following functions are continuous at the indicated point. Show the work that leads to your answer using correct notation and give a reason for your answer. Justify by sketching a graph of each function.

(a)  $f(x) = \begin{cases} 2x^2-3x, & x \leq -1 \\ 2-4x, & x > -1 \end{cases}$  at  $x = -1$

(b)  $g(x) = \begin{cases} \sqrt{5-x}, & x < 1 \\ 3x^2+1, & x \geq 1 \end{cases}$  at  $x = 1$

The graph of  $y = f(x)$  is given below. Answer questions 15-23 below.



15.  $\lim_{x \rightarrow 1^-} f(x) =$

16.  $\lim_{x \rightarrow 1^+} f(x) =$

17.  $\lim_{x \rightarrow 1} f(x) =$

18.  $f(1) =$

19.  $\lim_{x \rightarrow 4^-} f(x) =$

20.  $\lim_{x \rightarrow 4^+} f(x) =$

21.  $\lim_{x \rightarrow 4} f(x) =$

22.  $f(4) =$

23.  $\lim_{x \rightarrow \infty} f(x) =$

24. Draw the graph of a function  $f(x)$  on the interval  $-\infty < x \leq 7$  with the following characteristics.

(Answers will, and should, vary verily).

$$\lim_{x \rightarrow -10^-} f(x) = -2 = \lim_{x \rightarrow -10^+} f(x), \quad f(-10) = 2, \quad \lim_{x \rightarrow 0^-} f(x) = -\infty, \quad \lim_{x \rightarrow 0^+} f(x) = \infty$$

$$\lim_{x \rightarrow 5^-} f(x) = 0 = f(5), \quad \lim_{x \rightarrow 5^+} f(x) = 4, \quad f(7) = 4, \quad \lim_{x \rightarrow \infty} f(x) = 0$$