

I.

① $1 + \frac{\cot A}{\csc A} - \sin^2 A = \cos A (\cos A + 1)$

$$1 + \frac{\cos A}{\sin A} \cdot \sin A - \sin^2 A$$

$$1 - \sin^2 A + \cos A$$

$$\cos^2 A + \cos A$$

$$\cos A (\cos A + 1)$$

- done

② $\tan B + \cot B = \csc B \sec B$

$$\frac{\sin B}{\cos B} + \frac{\cos B}{\sin B}$$

$$\frac{\sin^2 B + \cos^2 B}{\sin B \cos B}$$

$$\frac{1}{\sin B \cos B}$$

$$\csc B \sec B$$

- done

③ $\frac{\cot \phi}{\sec \phi} = \csc \phi - \sin \phi$

$$\frac{1}{\sin \phi} - \frac{\sin \phi}{1 - \sin^2 \phi}$$

$$\frac{\cos^2 \phi}{\sin \phi}$$

$$\frac{\cos \phi}{\sin \phi} \cdot \frac{\cos \phi}{1}$$

$$\cot \phi (\sec \phi)$$

$$\frac{\cot \phi}{\sec \phi}$$

- done

④ $\frac{\sec \beta}{1 + \cos \beta} = \csc^2 \beta (\sec \beta - 1)$

$$= \frac{1}{\sin^2 \beta} \left(\frac{1}{\cos \beta} - 1 \right)$$

$$= \frac{1}{\sin^2 \beta \cos \beta} - \frac{1}{\sin^2 \beta} \left(\frac{\cos \beta}{\cos \beta} \right)$$

$$= \frac{1 - \cos \beta}{(\sin^2 \beta) \cos \beta}$$

$$= \frac{1 - \cos \beta}{(1 - \cos^2 \beta) \cos \beta}$$

$$= \frac{(1 - \cos \beta)}{(1 - \cos \beta)(1 + \cos \beta) \cos \beta}$$

$$= \frac{\sec \beta}{1 + \cos \beta}$$

- done

⑤ $(\tan \alpha - \sec \alpha)^2 = \frac{1 - \sin \alpha}{1 + \sin \alpha}$

$$\left(\frac{\sin \alpha}{\cos \alpha} - \frac{1}{\cos \alpha} \right)^2$$

$$\left(\frac{\sin \alpha - 1}{\cos \alpha} \right)^2$$

$$\frac{\sin^2 \alpha - 2 \sin \alpha + 1}{\cos^2 \alpha}$$

$$\frac{1 - 2 \sin \alpha + \sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$\frac{(1 - \sin \alpha)^2}{(1 - \sin \alpha)(1 + \sin \alpha)}$$

$$\frac{(1 - \sin \alpha)(1 - \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)}$$

$$\frac{1 - \sin \alpha}{1 + \sin \alpha}$$

- done

⑥ $\sin^4 \psi - \cos^4 \psi = 1 - 2 \cos^2 \psi$

$$(\sin^2 \psi - \cos^2 \psi)(\sin^2 \psi + \cos^2 \psi)$$

$$((1 - \cos^2 \psi) - \cos^2 \psi) \cdot 1$$

$$1 - 2 \cos^2 \psi$$

- done

⑦ $\sec^4 \delta - 2 \sec^2 \delta \tan^2 \delta + \tan^4 \delta = 1$

$$(\sec^2 \delta - \tan^2 \delta)^2$$

$$\left((1 + \tan^2 \delta) - \tan^2 \delta \right)^2$$

$$1^2$$

$$1$$

- done

⑧ $\sqrt[3]{\tan^2 x - \sec^2 x} = -1$
 $\sqrt[3]{(\sec^2 x - 1) - \sec^2 x}$
 $\sqrt[3]{-1}$
 -1 - done

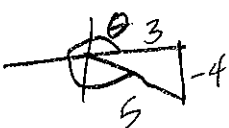
⑨ $\frac{1}{\csc y + \cot y} = \frac{1 - \cos y}{\sin y}$
 $\frac{1}{\frac{1}{\sin y} + \frac{\cos y}{\sin y}} = \frac{1 - \cos y}{\sin y} \left(\frac{1 + \cos y}{1 + \cos y} \right)$
 $\frac{1}{\frac{1 + \cos y}{\sin y}} = \frac{1 - \cos^2 y}{\sin y (1 + \cos y)}$
 $\frac{\sin y}{1 + \cos y} = \frac{\sin y \cdot \sin y}{\sin y (1 + \cos y)}$
 $\frac{\sin y}{1 + \cos y} = \frac{\sin y}{1 + \cos y}$ - done

⑩ $\frac{\sin k - 1}{\cos k} = \frac{\tan k - \sec k}{\cos k} = \frac{\frac{\sin k}{\cos k} - \frac{1}{\cos k}}{\cos k} = \frac{\sin k - 1}{\cos k}$ - done

⑪ $\frac{\sec w}{\sin w} - \frac{\sin w}{\cos w} = \cot w$
 $\frac{\sec w \cos w - \sin^2 w}{\sin w \cos w} = \frac{1 - \sin^2 w}{\sin w \cos w} = \frac{\cos^2 w - \sin^2 w}{\sin w \cdot \cos w} = \cot w$ - done

II. ⑫ $\sqrt{3} \cot x - 1 = 0, x \in [0, 2\pi)$
 $\cot x = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$
 $\tan x = \frac{\sqrt{3}}{2}$

III. ⑬ $\cos \theta = -\frac{7}{13}, \frac{\pi}{2} < \theta < \pi$
 $\sin \theta = \frac{2\sqrt{30}}{13}$
 $\csc \theta = \frac{13}{2\sqrt{30}} = \frac{13\sqrt{30}}{60}$
 $\sec \theta = -\frac{13}{7}$
 $\tan \theta = -\frac{2\sqrt{30}}{7}$
 $\cot \theta = -\frac{7}{2\sqrt{30}} = -\frac{7\sqrt{30}}{60}$

⑭ $\sin \theta = -\frac{4}{5}, \frac{3\pi}{2} < \theta < 2\pi$

 $\sin \theta = -\frac{4}{5} \Rightarrow \csc \theta = -\frac{5}{4}$
 $\cos \theta = \frac{3}{5} \Rightarrow \sec \theta = \frac{5}{3}$
 $\tan \theta = -\frac{4}{3} \Rightarrow \cot \theta = -\frac{3}{4}$

IV. ⑮ $\tan^2(-\pi) + \sec^2 \frac{3\pi}{4} = \sqrt{\cos^2(\frac{5\pi}{6}) + \sin^2(\frac{5\pi}{6})}$
 $0^2 + (-\sqrt{2})^2 = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2}$
 $2 \neq 1$

⑯ $\csc \frac{5\pi}{6} + \sec(-\frac{\pi}{3}) = \left[\cot^2(\frac{3\pi}{4}) + \tan^2(\frac{2\pi}{4}) \right]^2$
 $2 + \frac{1}{2} = [1 + 1]^2$
 $\frac{5}{2} \neq 4$

V. (17) $\frac{\cos^2 \theta - \sin^2 \theta}{(1 - \sin^2 \theta) - \sin^2 \theta} = \frac{1 - 2\sin^2 \theta}{1 - 2\sin^2 \theta}$ - done

(18) $\frac{(1 - \sin^2 \theta)(1 + \tan^2 \theta)}{\cos^2 \theta \cdot \sec^2 \theta} = 1$
 $\frac{(\cos \theta \cdot \sec \theta)^2}{1} = 1$ - done

(19) $\frac{\frac{1}{1 + \sin \phi} + \frac{1}{1 - \sin \phi}}{\frac{1 - \sin \phi + 1 + \sin \phi}{(1 + \sin \phi)(1 - \sin \phi)}} = 2 \sec^2 \phi$
 $\frac{2}{1 - \sin^2 \phi} = \frac{2}{\cos^2 \phi} = 2 \sec^2 \phi$ - done

(20) $\frac{\cos^2 \theta - \sin^2 \theta}{\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \cdot \frac{1 - \tan^2 \theta}{\sec^2 \theta}} = \frac{(1 - \tan^2 \theta) \cos^2 \theta}{\cos^2 \theta - \frac{\cos^2 \theta (\sin^2 \theta)}{\cos^2 \theta}} = \frac{(1 - \tan^2 \theta) \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta}$ - done

(21) $\frac{(\csc \beta - \sin \beta)^2}{\left(\frac{1}{\sin \beta} - \frac{\sin \beta}{1}\right)^2} = \frac{\cot^2 \beta - \cos^2 \beta}{\frac{\cos^2 \beta}{\sin^2 \beta} - \frac{\cos^2 \beta}{1}}$
 $\frac{\left(\frac{1 - \sin^2 \beta}{\sin \beta}\right)^2}{\left(\frac{\cos^2 \beta}{\sin \beta}\right)^2} = \frac{\cos^2 \beta - \cos^2 \beta \sin^2 \beta}{\sin^2 \beta}$
 $\frac{\cos^4 \beta}{\sin^2 \beta} = \frac{\cos^2 \beta (1 - \sin^2 \beta)}{\sin^2 \beta} = \frac{\cos^4 \beta}{\sin^2 \beta}$

(22) $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$
 $\frac{\cos^2 \theta + (1 - \sin \theta)^2}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta + 1 - 2\sin \theta + \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$
 $\frac{1 + 1 - 2\sin \theta}{\cos \theta (1 - \sin \theta)} = \frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)} = 2 \sec \theta$ - done

(23) $\frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$
 $\frac{\sec^2 x - 1}{\sec x + 1} = \frac{(1 - \cos x)(1 + \sec x)}{(1 + \sec x)}$
 $\frac{1}{\cos x} - 1 = \frac{1 - \cos x}{\cos x}$ - done

(24) $\frac{\cot x}{\csc x + 1} = \frac{\csc x - 1}{\cot x} \cdot \frac{(\csc x + 1)}{(\csc x + 1)}$
 $\frac{\csc^2 x - 1}{\cot x (\csc x + 1)} = \frac{\cot x \cdot \cot x}{\cot x (\csc x + 1)} = \frac{\cot x}{\csc x + 1}$ - done

VI. (25) $\sin^2 \theta + \cos 2\theta$
 $\frac{1}{2}(1 - \cos 2\theta) + \cos 2\theta$
 $\frac{1}{2} - \frac{1}{2}\cos 2\theta + \cos 2\theta$
 $\frac{1}{2} + \frac{1}{2}\cos 2\theta$

(27) $\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$
 $\frac{1}{\cos^2 \theta}$
 $\sec^2 \theta$

(26) $\sec^4 x - \tan^4 x - \tan^2 x$
 $(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x) - \tan^2 x$
 $1 \cdot (\sec^2 x + \tan^2 x) - \tan^2 x$
 $\sec^2 x + 0$
 $\sec^2 x$

(28) $\frac{1 + \tan \phi}{\sin \phi} - \sec \phi$

$\left(\frac{\cos \phi}{\cos \phi}\right) \frac{1 + \sin \phi}{\cos \phi} - \frac{1}{\cos \phi}$
 $\frac{\cos \phi + \sin \phi}{\sin \phi \cos \phi} - \frac{1}{\cos \phi} \left(\frac{\sin \phi}{\sin \phi}\right)$
 $\frac{\cos \phi + \sin \phi - \sin \phi}{\sin \phi \cos \phi}$
 $\frac{\cos \phi}{\sin \phi \cos \phi}$
 $\csc \phi$

VII.

(29) $\cos 2\theta = 2 \cos \theta$, $\theta = \frac{\pi}{2}$
 $\cos(2(\frac{\pi}{2})) = 2 \cos \frac{\pi}{2}$
 $\cos \pi = 2(0)$
 $-1 \neq 0$

(30) $\sin(\theta - \beta) = \sin \theta - \sin \beta$, $\theta = \frac{\pi}{2}$, $\beta = \frac{\pi}{4}$
 $\sin(\frac{\pi}{2} - \frac{\pi}{4}) = \sin \frac{\pi}{2} - \sin \frac{\pi}{4}$
 $\sin \frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2}$
 $\frac{\sqrt{2}}{2} \neq \frac{2 - \sqrt{2}}{2}$

(31) $\tan \frac{1}{2} \theta = \frac{1}{2} \tan \theta$, $\theta = \frac{\pi}{2}$
 $\tan(\frac{1}{2}(\frac{\pi}{2})) = \frac{1}{2} \tan \frac{\pi}{2}$
 $\tan \frac{\pi}{4} = \text{DNE}$
 $1 \neq$

(32) $\sqrt{\tan^2 \theta + 1} = \sec \theta$ (Choose an angle in QII or QIII)
 $\sqrt{(\tan \frac{2\pi}{3})^2 + 1} = \sec \frac{2\pi}{3}$ where $\sec \theta < 0$
 $\sqrt{(-\sqrt{3})^2 + 1} = -2$
 $\sqrt{3 + 1} = 2 \neq$