Name KEY Date________ Period______

Worksheet 1.3—Simplifying & Evaluating

Show all work. Give simplified, exact values for all answers. **No Calculator**

I. Multiple Choice

C 1. Evaluate $16^{3/4}$

(A) $\frac{1}{32}$ (B) $\frac{1}{8}$ (C) 8 (D) 32 (E) 64

$16^{3/4}$
$(16^{1/4})^3$
$(\sqrt[4]{16})^3$
2^3
8

A 2. Simplify the radical $\sqrt[3]{40}$

(A) $2\sqrt[3]{5}$ (B) $8\sqrt[3]{5}$ (C) 15 (D) 20 (E) $16\sqrt[3]{5}$

$\sqrt[3]{40}$
$\sqrt[3]{8 \cdot 5}$
$\sqrt[3]{8} \cdot \sqrt[3]{5}$
$2\sqrt[3]{5}$

C 3. Simplify completely: $3\sqrt{5}\left(2\sqrt{10}-2\sqrt{5}\right)$

(A) $6\sqrt{2}-30$ (B) $30\sqrt{2}-6$ (C) $30\sqrt{2}-30$ (D) 10 (E) 30

$3\sqrt{5}(2\sqrt{10} - 2\sqrt{5})$
$6\sqrt{50} - 6 \cdot 5$
$6\sqrt{25 \cdot 2} - 30$
$30\sqrt{2} - 30$

E 4. Simplify: $2\sqrt[3]{24}-5\sqrt[3]{375}$

(A) $-19\sqrt{3}$ (B) $-21\sqrt{15}$ (C) $-21\sqrt[3]{15}$ (D) $-3\sqrt[3]{351}$ (E) $-21\sqrt[3]{3}$

$2\sqrt[3]{8 \cdot 3} - 5\sqrt[3]{3 \cdot 125}$
$2 \cdot 2\sqrt[3]{3} - 5 \cdot 5\sqrt[3]{3}$
$4\sqrt[3]{3} - 25\sqrt[3]{3}$
$-21\sqrt[3]{3}$

A 5. Which of the following is equivalent to $\sqrt[3]{\frac{1}{2}}$?

(A) $\frac{\sqrt[3]{4}}{2}$ (B) $\frac{\sqrt[3]{2}}{2}$ (C) $\frac{\sqrt{4}}{2}$ (D) $2\sqrt[3]{4}$ (E) $\sqrt{\frac{1}{8}}$

$\sqrt[3]{\frac{1}{2}}$

$\frac{\sqrt[3]{1}}{\sqrt[3]{2}}$

$\frac{1}{\sqrt[3]{2}}$

$\frac{1}{2^{1/3}} \cdot \frac{2^{2/3}}{2^{2/3}}$

$\frac{2^{2/3}}{2}$

$\frac{(2^2)^{1/3}}{2}$

$\frac{\sqrt[3]{4}}{2}$

B 6. If $f(x)=\frac{2}{x}$, evaluate $f\left(1-\sqrt{3}\right)$. Rationalize your answer.

(A) $2\sqrt{3}+1$ (B) $-1-\sqrt{3}$ (C) $2-\sqrt{3}$ (D) $\sqrt{3}-1$ (E) $2\sqrt{3}$

$f(1-\sqrt{3}) = \frac{2}{1-\sqrt{3}}$

$= \frac{2}{1-\sqrt{3}}\left(\frac{1+\sqrt{3}}{1+\sqrt{3}}\right)$

$= \frac{2+2\sqrt{3}}{1-3}$

$= \frac{2}{-2} + \frac{2\sqrt{3}}{-2}$

$= -1-\sqrt{3}$

C 7. Simplify the complex fraction: $\dfrac{\frac{x}{5}}{\frac{4}{x+6}}$

(A) $\frac{5(x+6)}{4x}$ (B) $20x(x+6)$ (C) $\frac{x(x+6)}{20}$ (D) $\frac{x+6}{20x}$ (E) $\frac{4x}{5(x+6)}$

LCM
LCM

$\dfrac{\left(\frac{x}{5}\right)}{\left(\frac{4}{x+6}\right)} \cdot \frac{5(x+6)}{5(x+6)}$

$\frac{x(x+6)}{4 \cdot 5}$

$\frac{x(x+6)}{20}$

C 8. Simplify the complex fraction: $\dfrac{x+\frac{9}{x}}{x+\frac{x}{9}}$

(A) $\frac{x^2+9}{10x}$ (B) 1 (C) $\frac{9x^2+81}{10x^2}$ (D) $\frac{9x^2+81}{9x^2+x}$ (E) $\frac{x^2+9}{x+9}$

LCM
LCM

$\dfrac{x+\frac{9}{x}}{x+\frac{x}{9}}\left(\frac{9x}{9x}\right)$

$\frac{9x^2+81}{9x^2+x^2}$

$\frac{9x^2+81}{10x^2}$

E 9. Simplify the complex fraction: $\dfrac{8x^{-1}-5x^{-2}}{5x^{-3}-8x^{-1}}$

(A) $\dfrac{5+8x^2}{5-8x^2}$ (B) $\dfrac{8x-5}{5-8x^2}$ (C) $\dfrac{8x-5}{5-8x}$ (D) $\dfrac{8x^2-5x}{5+8x^2}$ (E) $\dfrac{8x^2-5x}{5-8x^2}$

$$\frac{8x^{-1}-5x^{-2}}{5x^{-3}-8x^{-1}}$$

$$\frac{\left(\frac{8}{x}-\frac{5}{x^2}\right)}{\left(\frac{5}{x^3}-\frac{8}{x}\right)}\left(\frac{x^3}{x^3}\right)$$

$$\frac{8x^2-5x}{5-8x^2}$$

E 10. Simplify the complex fraction: $\dfrac{\dfrac{x}{x-1}+\dfrac{3}{x+5}}{\dfrac{5}{x+5}-\dfrac{x}{x-1}}$

(A) $\dfrac{x^2+8x-3}{x^2+10x-5}$ (B) $\dfrac{x^2+2x+15}{x^2+10x-5}$ (C) $\dfrac{x^2+2x+15}{-x^2-5}$ (D) $\dfrac{x^2+2x+15}{-x^2-5}$ (E) $\dfrac{x^2+8x-3}{-x^2-5}$

$$\frac{\left(\frac{x}{x-1}+\frac{3}{x+5}\right)}{\left(\frac{5}{x+5}-\frac{x}{x-1}\right)}\left(\frac{(x+5)(x-1)}{(x+5)(x-1)}\right)$$

$$\frac{x(x+5)+3(x-1)}{5(x-1)-x(x+5)}$$

$$\frac{x^2+5x+3x-3}{5x-5-x^2-5x}$$

$$\frac{x^2+8x-3}{-x^2-5}$$

B 11. If $f(x)=5x^2-2x+3$, what is the simplified expresssion for $\dfrac{f(x+p)-f(x-p)}{2p}$ for some constant $p\neq 0$?

(A) $20x-4$ (B) $10x-2$ (C) $40x-8$ (D) $5x-2$ (E) $\dfrac{5}{2}x-1$

$$\frac{f(x+p)-f(x-p)}{2p}$$

$$\frac{\left[5(x+p)^2-2(x+p)+3\right]-\left[5(x-p)^2-2(x-p)+3\right]}{2p}$$

$$\frac{5x^2+10xp+5p^2-2x-2p+3-5x^2+10xp-5p^2+2x-2p-3}{2p}$$

$$\frac{20xp-4p}{2p}$$

$$\frac{20xp}{2p}-\frac{4p}{2p}$$

$$10x-2$$

II. Short Answer

12. Evaluate each of the following.

(a) $8^{\frac{2}{3}}$

$(\sqrt[3]{8})^2$

2^2

4

(b) $8^{-\frac{2}{3}}$

$\frac{1}{8^{2/3}}$

$\frac{1}{4}$

(c) $-8^{\frac{2}{3}}$

$(-1)8^{2/3}$

-4

(d) $(-8)^{\frac{2}{3}}$

$(\sqrt[3]{-8})^2$

$(-2)^2$

4

13. Simplify the following, if possible. Put your answer in radical form.

(a) $\sqrt{8}$

$\sqrt{4 \cdot 2}$

$2\sqrt{2}$

(b) $\sqrt{75}$

$\sqrt{25 \cdot 3}$

$5\sqrt{3}$

(c) $\sqrt[3]{24}+\sqrt[3]{81}$

$\sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$

$2\sqrt[3]{3} + 3\sqrt[3]{3}$

$5\sqrt[3]{3}$

(d) $\sqrt[4]{16x^8y^{15}}$

$\sqrt[4]{16}\, x^{8/4}\, y^{15/4}$

$2x^2\sqrt[4]{y^4y^4y^4y^3}$

$2x^2y^3\sqrt[4]{y^3}$

(e) $(x^2+4)^{1/2}$

$\sqrt{x^2+4}$ (done)

14. Rationalize and simplify each of the following by clearing the radical(s) from either the numerator or denominator. Leave your final answer in radical form when necessary.

(a) $\frac{3}{\sqrt{5}}$

$\frac{3}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$

$\frac{3\sqrt{5}}{5}$

(b) $\frac{3\sqrt{2}}{4}$

$\frac{3\sqrt{2}}{4}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\frac{3 \cdot 2}{4\sqrt{2}}$

$\frac{3}{2\sqrt{2}}$

(c) $\frac{2}{\sqrt{x}+\sqrt{y}}$

$\frac{2}{\sqrt{x}+\sqrt{y}}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)$

$\frac{2(\sqrt{x}-\sqrt{y})}{x-y}$

(d) $\frac{t^2-4}{\sqrt{t+2}-2}$

$\frac{(t-2)(t+2)}{\sqrt{t+2}-2}\left(\frac{\sqrt{t+2}+2}{\sqrt{t+2}+2}\right)$

$\frac{(t-2)(t+2)(\sqrt{t+2}+2)}{t+2-4}$

$\frac{\cancel{(t-2)}(t+2)(\sqrt{t+2}+2)}{\cancel{(t-2)}}$

$(t+2)(\sqrt{t+2}+2)$

(e) $\frac{4\sqrt[4]{3}}{5}$

$\frac{4 \cdot 3^{1/4}}{5}\left(\frac{3^{3/4}}{3^{3/4}}\right)$

$\frac{4 \cdot 3}{5\sqrt[4]{3^3}}$

$\frac{12}{5\sqrt[4]{27}}$

15. If $f(x) = 2 - x^2$ evaluate the following. Simplify if possible.

a) $f^2(-1)$

$f^2(-1)$

$[f(-1)]^2$

$[2-(-1)^2]^2$

$[2-1]^2$

1^2

1

b) $f(2x-1)$

$f(2x-1)$

$2-(2x-1)^2$

$2-(4x^2-4x+1)$

$2-4x^2+4x-1$

$-4x^2+4x+1$

c) $(f \circ f)(x)$

$(f \circ f)(x)$

$f(f(x))$

$f(2-x^2)$

$2-(2-x^2)^2$

$2-(4-4x^2+x^4)$

$2-4+4x^2-x^4$

$-x^4+4x^2-2$

d) $f(f(f(1)))$

$f(f(2-(1)^2))$

$f(f(2-1))$

$f(f(1))$

$f(2-(1)^2)$

$f(1)$

$2-1^2$

1

e) $\dfrac{f(x+p)-f(x)}{p}$

$\dfrac{[2-(x+p)^2]-[2-x^2]}{p}$

$\dfrac{2-x^2-2xp-p^2-2+x^2}{p}$

$\dfrac{-2xp-p^2}{p}$

$\dfrac{\cancel{p}(-2x-p)}{\cancel{p}}$

$-2x-p$

16. Simplify each of the following rational expressions to a single term. State any prohibited *x*-values in your final answer.

(a) $\dfrac{2x^2-8}{x^2-4x+4}$

$\dfrac{2(x^2-4)}{(x-2)^2}$

$\dfrac{2\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-2)}$

$\dfrac{2(x+2)}{x-2}$, $x \neq 2$

(b) $\dfrac{x^2-5x-6}{6x-x^2}$

$\dfrac{(x-6)(x+1)}{x(6-x)}$

$\dfrac{\cancel{(x-6)}(x+1)}{-x\cancel{(x-6)}}$

$\dfrac{x+1}{-x}$, $x \neq 0, 6$

(c) $\dfrac{x-4}{x^2-4} \div \dfrac{x^2-3x-4}{x^2+5x+6}$

$\dfrac{\cancel{(x-4)}}{(x-2)\cancel{(x+2)}} \cdot \dfrac{(x+3)\cancel{(x+2)}}{\cancel{(x-4)}(x+1)}$

$\dfrac{x+3}{(x-2)(x+1)}$, $x \neq 2, -1, -2, 4$

(d) $\dfrac{\dfrac{x}{y}+\dfrac{1}{x^3}}{1-\dfrac{y}{x}}$

$\dfrac{\left(\dfrac{x}{y}+\dfrac{1}{x^3}\right)}{\left(\dfrac{1}{1}-\dfrac{y}{x}\right)}\left(\dfrac{x^3y}{x^3y}\right)$

$\dfrac{x^4+y}{x^3y-x^2y^2}$, $x, y \neq 0$

(e) $\dfrac{(1+x^2)^{1/2}-x^2(1+x^2)^{-1/2}}{1+x^2}$

$\dfrac{\left(\dfrac{\sqrt{1+x^2}}{1}-\dfrac{x^2}{\sqrt{1+x^2}}\right)}{(1+x^2)}\left(\dfrac{\sqrt{1+x^2}}{(1+x^2)^{1/2}}\right)$

$\dfrac{1+x^2-x^2}{(1+x^2)^{3/2}}$

$\dfrac{1}{\sqrt{(1+x^2)^3}}$