Name____ Date_______________________ Period______

Worksheet 2.5—Building Functions from other Functions

Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

__A__ 1. If the point $(3,4)$ lies on the graph of an invertible function *f*, then which of the following points lies on the graph of its inverse?

(A) $(4,3)$ (B) $(3,-4)$ (C) $\left(3,\frac{1}{4}\right)$ (D) $(-3,4)$ (E) None of these

__A__ 2. The inverse of the function $f(x)=7x+8$ will be

(A) $g(x)=\frac{x-8}{7}$ (B) $g(x)=\frac{1}{7x+8}$ (C) $g(x)=\frac{8}{x-7}$ (D) $g(x)=-7x-8$ (E) $g(x)=-\frac{1}{7}x+8$

$y=7x+8$
$x=7y+8$
$\frac{x-8}{7}=\frac{7y}{7}$

$g(x)=\frac{x-8}{7}$

__C__ 3. If $f(x)=\sqrt{x}$ and $g(x)=x^2$, then $(gf)(x)=$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

$(gf)(x)=\sqrt{x}\cdot x^2$
$=x^{1/2}x^2$
$=x^{1/2+2}$
$=x^{5/2}$

D 4. If $f(x)=\sqrt{x}$ and $g(x)=x^2$, then $(g\circ f)(x)=$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

$(g\circ f)(x)=(\sqrt{x})^2$
$=(x^{1/2})^2$
$=x$

D 5. If $f(x)=\sqrt{x}$ and $g(x)=x^2$, then $(f\circ g)(x)=$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

$(f\circ g)(x)=(x^2)^{1/2}$
$=x$

C 6. Suppose f and g are functions with domain of all real numbers. Which of the following is NOT necessarily true?

$f+g=g+f$ $f\cdot g=g\cdot f$

(A) $(f+g)(x)=(g+f)(x)$ (B) $(fg)(x)=(gf)(x)$ (C) $f(g(x))=g(f(x))$

(D) $(f-g)(x)=-(g-f)(x)$ (E) $(f\circ g)(x)=f(g(x))$

$=-(-f+g)$ same
$=(f-g)$

A 7. If $f(x)=x-7$ and $g(x)=\sqrt{4-x}$, what is the domain of $\frac{f}{g}$?

(A) $(-\infty,4)$ (B) $(-\infty,4]$ (C) $(4,\infty)$ (D) $[4,\infty)$ (E) $(4,7)\cup(7,\infty)$

$\frac{f}{g}=\frac{x-7}{\sqrt{4-x}}$ $4-x>0$
$-x>-4$
$x<4$

E 8. If $f(x)=x^2+1$, then $(f\circ f)(x)=$

(A) $2x^2+2$ (B) $2x^2+1$ (C) x^4+1 (D) x^4+2x^2+1 (E) x^4+2x^2+2

$(f\circ f)(x)=(x^2+1)^2+1$
$=x^4+2x^2+1+1$
$=x^4+2x^2+2$

B 9. Which of the following relations is equivalent to $y=|x|$?

(A) $y=x$ (B) $y=\sqrt{x^2}$ (C) $y^3=x^3$ (D) $y=(\sqrt{x})^2$ (E) $x=|y|$

E 10. Let $h(x)=\dfrac{4x+5}{2x-7}$ and $f(x)=x+6$. If $h(x)=(g\circ f)(x)$, then $g(x)$ is ??

(A) $\dfrac{4x+1}{2x-13}$ (B) $\dfrac{4x-1}{2x+13}$ (C) $\dfrac{4x}{2x}-\dfrac{5}{7}$ (D) $\dfrac{4x-19}{2x-5}$ (E) None of these

$h(x)=\dfrac{4(x+6)+1}{2(x+6)-13}=\dfrac{4x+24+1}{2x+12-13}$

$=\dfrac{4(x+6)-1}{2(x+6)-1}=\dfrac{4x+24-1}{2x+12-1}$

$=\dfrac{4(x+6)}{2(x+6)}-\dfrac{5}{7}=\dfrac{(7)\,4x+24}{(7)\,2x+12}-\dfrac{5\,(2x+12)}{7\,(2x+12)}=\dfrac{28x+168}{14x+84}-\dfrac{10x+60}{14x+84}$

$=\dfrac{4(x+6)-19}{2(x+6)-5}=\dfrac{4x+24-19}{2x+12-5}=\dfrac{4x+5}{2x+7}$

II. Short Answer

11. If $f(x)=\sqrt{x+3}$ and $g(x)=\sqrt{x-4}$, find formulas for $h=:\ \dfrac{f}{g},\ \dfrac{g}{f},\ f+g,\ f\circ g$, and $g\circ f$.

Give the domain of each.

$\dfrac{f}{g}=\dfrac{\sqrt{x+3}}{\sqrt{x-4}}$

$x+3\geq 0$ $x-4\geq 0$ $\sqrt{x-4}\neq 0$

$x\geq -3$ $x\geq 4$ $x\neq 4$

so $x>4$

$D_f: \{x \mid x>4\}$

$\dfrac{g}{f}=\dfrac{\sqrt{x-4}}{\sqrt{x+3}}$

$x-4\geq 0$ $x+3\geq 0$ $\sqrt{x+3}\neq 0$

$x\geq 4$ $x\geq -3$ $x\neq -3$

$\{x \mid x\geq 4\}$

$f+g=\sqrt{x+3}+\sqrt{x-4}$

$x+3\geq 0$ $x-4\geq 0$

$x\geq -3$ $x\geq 4$

$\{x \mid x\geq 4\}$

$f\circ g=\sqrt{\sqrt{x-4}+3}$

$x-4\geq 0$ $\sqrt{x-4}+3\geq 0$

$x\geq 4$ $\sqrt{x-4}\geq -3$

$D: \{x \mid x\geq 4\}$

$g\circ f=\sqrt{\sqrt{x+3}-4}$

$x+3\geq 0$ $\sqrt{x+3}-4\geq 0$

$x\geq -3$ $\sqrt{x+3}\geq 4$

$x+3\geq 16$

$x\geq 13$

$D: \{x \mid x\geq 13\}$

12. For each of the following, find $f(g(x))$ and $g(f(x))$. Find the domain of each and decide if *f* and *g* are inverses. Give an explanation for your answers.

(a) $f(x)=\dfrac{1}{x-1}$, $g(x)=\sqrt{x}$

$f(g(x))=\dfrac{1}{\sqrt{x}-1}$

$x\geq 0$ $\sqrt{x}-1\neq 0$

$\sqrt{x}\neq 1$

$x\neq 1$

$D: \{x \mid x\geq 0, x\neq 1\}$

$g(f(x))=\sqrt{\dfrac{1}{x-1}}=\dfrac{\sqrt{1}}{\sqrt{x-1}}$

$\sqrt{x-1}\neq 0$ $x-1\geq 0$

$x\neq 1$ $x\geq 1$

$D: \{x \mid x>1\}$

$f(g(x))=\dfrac{1}{\sqrt{x}-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\dfrac{\sqrt{x}+1}{x-1}$

$g(f(x))=\dfrac{1}{\sqrt{x-1}}\cdot\dfrac{\sqrt{x-1}}{\sqrt{x-1}}=\dfrac{\sqrt{x-1}}{x-1}$

f+g are not inverses

(b) $f(x)=\dfrac{1}{x+1}$, $g(x)=\dfrac{1}{x-1}$

$f(g(x))=\dfrac{1}{\frac{1}{x-1}+1}$

$x-1\neq 0$ $\frac{1}{x-1}+1\neq 0$

$x\neq 1$ $\frac{1}{x-1}\neq -1$

$1\neq -(x-1)$

$1\neq -x+1$

$0\neq -x$

$x\neq 0$

$D: \{x \mid x\neq 0, 1\}$

$f(g(x))=\dfrac{1}{\frac{1}{x-1}+1}\cdot\dfrac{(x-1)}{(x-1)}=\dfrac{x-1}{1+x-1}=\dfrac{x-1}{x}$

$g(f(x))=\dfrac{1}{\frac{1}{x+1}-1}$

$x+1\neq 0$ $\frac{1}{x+1}-1\neq 0$

$x\neq -1$ $\frac{1}{x+1}\neq 1$

$1\neq x+1$

$0\neq x$

$D: \{x \mid x\neq 0, -1\}$

$g(f(x))=\dfrac{1}{\frac{1}{x+1}-1}\cdot\dfrac{(x+1)}{(x+1)}=\dfrac{x+1}{1-x-1}=\dfrac{x+1}{-x}$

f+g are not inverses

13. Decompose each of the following functions *h* into two functions *f* and *g* such that $h(x)=f(g(x))$. Find two, different, non-trivial decompositions.

(a) $h(x)=\sqrt{x^2-5x}$ (b) $h(x)=\dfrac{3}{x^3-5x+6}$ (c) $h(x)=\sqrt{x+e^{\sqrt{x}}}$

Answers will vary

14. Assume f is a one-to-one function.

(a) If $f(2)=9$, find $f^{-1}(9)$

$(2,9)$ $(9,2)$

$f^{-1}(9)=2$

(b) If $f^{-1}(-3)=1$, find $f(1)$

$(-3,1)$ $(1,3)$

$f(1)=3$

(c) if $f(x)=5-2x$, find $f^{-1}(-3)$

$y=5-2x$
$x=5-2y$
$x-5=-2y$
$y=\frac{x-5}{-2}$
$f^{-1}(x)=\frac{x-5}{-2}$

$f^{-1}(-3)=\frac{-3-5}{-2}$
$=\frac{-8}{-2}$
$=4$

15. Find the inverse, $g(x)$, of the following functions, then compose the functions to verify.

(a) $f(x)=\left(2-x^3\right)^5$

$y=(2-x^3)^5$
$(x)^{1/5}=\left((2-y^3)^5\right)^{1/5}$
$x^{1/5}=2-y^3$
$x^{1/5}-2=-y^3$
$\left(-x^{1/5}+2\right)^{1/3}=\left(y^3\right)^{1/3}$
$\left(-x^{1/5}+2\right)^{1/3}=g(x)$

$f(g(x))=\left(2-\left(\left(-x^{1/5}+2\right)^{1/3}\right)^3\right)^5$
$=\left(2-\left(-x^{1/5}+2\right)\right)^5$
$=\left(2+x^{1/5}-2\right)^5$
$=\left(x^{1/5}\right)^5$
$=x$

And

$g(f(x))=\left(-\left(\left(2-x^3\right)^5\right)^{1/5}+2\right)^{1/3}$
$=\left(-(2-x^3)+2\right)^{1/3}$
$=\left(-2+x^3+2\right)^{1/3}$
$=\left(x^3\right)^{1/3}$
$=x$

(b) $f(x)=\dfrac{2-7x}{3x-1}$

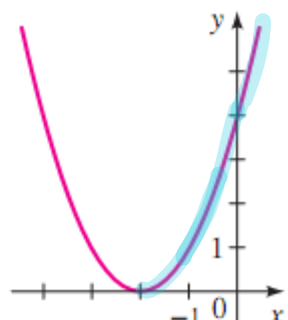
$y=\frac{2-7x}{3x-1}$
$x=\frac{2-7y}{3y-1}$
$x(3y-1)=2-7y$
$3xy-x=2-7y$
$3xy+7y=2+x$
$\frac{y(3x+7)}{3x+7}=\frac{2+x}{3x+7}$
$y=\frac{2+x}{3x+7}$
$g(x)=\frac{2+x}{3x+7}$

$f(g(x))=\dfrac{2-7\left(\frac{2+x}{3x+7}\right)}{3\left(\frac{2+x}{3x+7}\right)-1}\cdot\frac{(3x+7)}{(3x+7)}$
$=\frac{2(3x+7)-7(2+x)}{3(2+x)-(3x+7)}$
$=\frac{6x+14-14-7x}{6+3x-3x-7}$
$=\frac{-x}{-1}$
x

And $g(f(x))=\dfrac{2+\left(\frac{2-7x}{3x-1}\right)}{3\left(\frac{2-7x}{3x-1}\right)+7}\cdot\frac{(3x-1)}{(3x-1)}$
$=\frac{2(3x-1)+2-7x}{3(2-7x)+7(3x-1)}$
$=\frac{6x-2+2-7x}{6-21x+21x-7}$
$=\frac{-x}{-1}$
$=x$

16. The following functions are not one-to-one. Restrict each's domain so that the resulting function IS one-to-one. Write an equation for each graph (assume no dilations), then find the equation of the inverse function under the restricted domain.

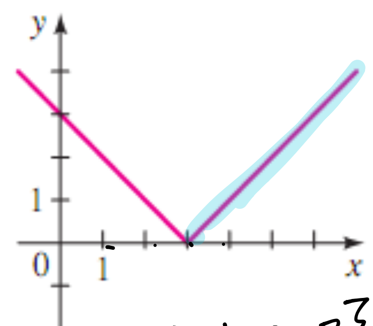
(a)



D: $\{x \mid x \geq -2\}$

$f(x)=(x+2)^2$
$y=(x+2)^2$
$\sqrt{x}=\sqrt{(y+2)^2}$
$\sqrt{x}=y+2$
$\sqrt{x}-2=y$
$f^{-1}(x)=\sqrt{x}-2$

(b)

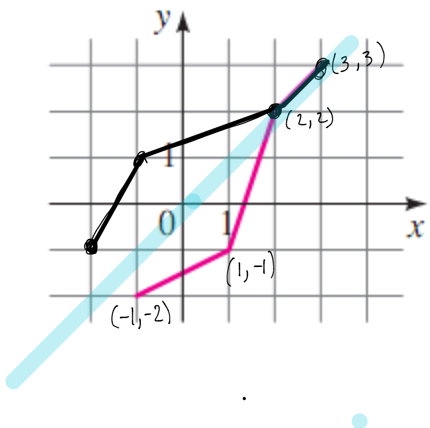


D: $\{x \mid x \geq 3\}$

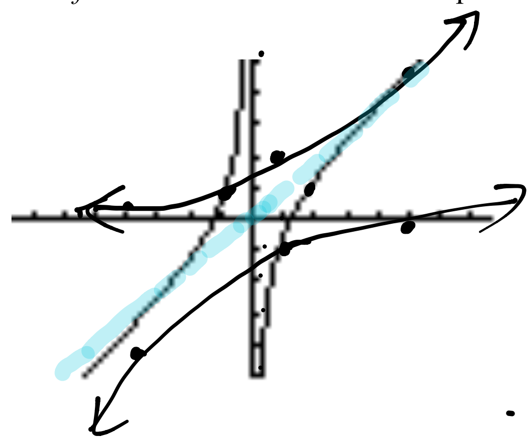
$f(x)=|x-3|$
$y=|x-3|$
$x=|y-3|$
$x=y-3$
$x+3=y$
$f^{-1}(x)=x+3$

17. Use the graph of each function, *f*, to sketch the graph of f^{-1}. Assume the scales are square.

(a)



(b)



18. Korpicello's Pizza charges a base price of \$5 for a large pizza, plus \$2 for each topping.

a. Write and equation for the total cost, *C*, of a large pizza with *n* toppings.

b. Find the equation for $C^{-1}(n)$, the inverse function of $C(n)$.

c. What is practical interpretation (or what is the usefulness) of $C^{-1}(n)$?

d. What are ***your*** favorite toppings? If you only had \$10 to spend, how many, and which, toppings would you/could you get?

a) $C(n)=2n+5$

$C=2n+5$

$n=2c+5$

$\frac{n-5}{2}=\frac{2c}{2}$

$c=\frac{n-5}{2}$

b) $C^{-1}(n)=\frac{n-5}{2}$

c) To find how many toppings you could buy.

D) $C^{-1}(10)=\frac{10-5}{2}$

$=\frac{5}{2}$ toppings