Name KEY Date Period

Worksheet 7.1—Polar Coordinates

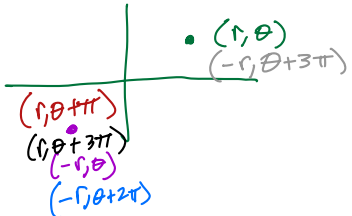
Show all work on a separate sheet of paper. Calculator permitted, show all set ups.

Multiple Choice

C

1. If $r \neq 0$, which of the following polar coordinate pairs represents the same point as the point with polar coordinates (r,θ)?

(A) $(-r,\theta)$ (B) $(-r,\theta+2\pi)$ (C) $(-r,\theta+3\pi)$ (D) $(r,\theta+\pi)$ (E) $(r,\theta+3\pi)$



C

2. Which of the following are the rectangular coordinates of the point with polar coordinate $\left(-2,-\frac{\pi}{3}\right)$?

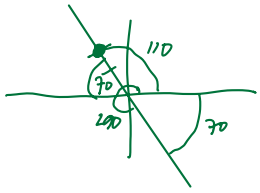
(A) $\left(-\sqrt{3},1\right)$ (B) $\left(-1,-\sqrt{3}\right)$ (C) $\left(-1,\sqrt{3}\right)$ (D) $\left(1,-\sqrt{3}\right)$ (E) $\left(1,\sqrt{3}\right)$

$(x,y) = (r\cos\theta, r\sin\theta)$
$= \left(-2\cos\left(-\frac{\pi}{3}\right), -2\sin\left(-\frac{\pi}{3}\right)\right)$
$= \left(-2\left(\frac{1}{2}\right), -2\left(-\frac{\sqrt{3}}{2}\right)\right)$
$= (-1, \sqrt{3})$

A

3. Which of the following polar coordinate pairs represent the same point as the point with polar coordinates $\left(2,110^\circ\right)$?

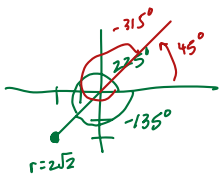
(A) $\left(-2,-70^\circ\right)$ (B) $\left(-2,110^\circ\right)$ (C) $\left(-2,-250^\circ\right)$ (D) $\left(2,-70^\circ\right)$ (E) $\left(2,290^\circ\right)$



E

4. Which of the following polar coordinate pairs does NOT represent the point with rectangular coordinates $(-2,-2)$?

(A) $\left(2\sqrt{2},-135^\circ\right)$ (B) $\left(2\sqrt{2},225^\circ\right)$ (C) $\left(-2\sqrt{2},-315^\circ\right)$ (D) $\left(-2\sqrt{2},45^\circ\right)$ (E) $\left(-2\sqrt{2},135^\circ\right)$



$r = \sqrt{4+4}$
$r = 2\sqrt{2}$

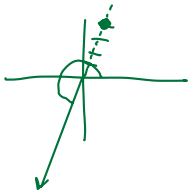
$\theta = \tan^{-1}\frac{-2}{-2}$
$\theta = 45^\circ$

Short Answer

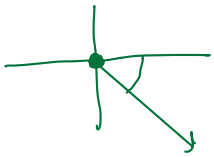
For problems 5-8, plot the point that has the given polar coordinates.

5. $\left(3,\frac{3\pi}{4}\right)$

6. $\left(-4,\frac{4\pi}{3}\right)$



7. $(0,-1)$



8. $\left(-2,-\frac{17\pi}{6}\right)$

$-\frac{17\pi}{6}+\frac{2\pi}{2}=\frac{7\pi}{6}$

$-\frac{17\pi}{6}\equiv\frac{7\pi}{6}$

For problems 9-12, plot the point that has the given polar coordinates, then give two other polar coordinate representations of the point, one with $r<0$ and the other with $r>0$.

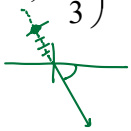
9. $\left(3,-\frac{\pi}{2}\right)$

$\left(3,\frac{3\pi}{2}\right)$ or $\left(-3,\frac{\pi}{2}\right)$

10. $\left(-2,\frac{7\pi}{6}\right)$

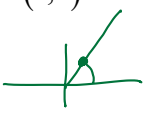
$\left(2,\frac{\pi}{6}\right)$ or $\left(-2,\frac{5\pi}{6}\right)$

11. $\left(-4,-\frac{\pi}{3}\right)$



$\left(4,\frac{2\pi}{3}\right)$ or $\left(-4,\frac{5\pi}{3}\right)$

12. $(1,1)$



$(-1,1+\pi)$ or $(1,-2\pi+1)$

$\approx(-1,4.14)$ or $(1,-5.28)$

For problems 13-16, find the rectangular coordinates for the point whose polar coordinates are given.

13. $\left(7,\frac{\pi}{6}\right)$

$\left(7\cos\frac{\pi}{6},7\sin\frac{\pi}{6}\right)$

$\left(7\left(\frac{\sqrt{3}}{2}\right),7\left(\frac{1}{2}\right)\right)$

$\left(\frac{7\sqrt{3}}{2},\frac{7}{2}\right)$

14. $\left(\sqrt{2},-\frac{3\pi}{4}\right)$

$\left(\sqrt{2}\cos\left(-\frac{3\pi}{4}\right),\sqrt{2}\sin\left(-\frac{3\pi}{4}\right)\right)$

$\left(\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right),\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$(-1,-1)$

15. $(7,7\pi)$

$(7\cos7\pi,7\sin7\pi)$

$(7(-1),7(0))$

$(-7,0)$

16. $\left(-\sqrt{3},-\frac{5\pi}{3}\right)$

$\left(-\sqrt{3}\cos\left(-\frac{5\pi}{3}\right),-\sqrt{3}\sin\left(-\frac{5\pi}{3}\right)\right)$

$\left(-\sqrt{3}\left(\frac{1}{2}\right),-\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\left(-\frac{\sqrt{3}}{2},-\frac{3}{2}\right)$

For problems 17-20, convert the rectangular coordinates to polar coordinates with $r>0$ and $0\le\theta<2\pi$.

17. $(-6,6)$

$r=\sqrt{36+36}$

$r=\sqrt{72}$

$r=6\sqrt{2}$

$\theta=\tan^{-1}\left(\frac{6}{-6}\right)$

$\theta=\frac{3\pi}{4}$

$\left(6\sqrt{2},\frac{3\pi}{4}\right)$

18. $\left(\sqrt{8},-\sqrt{8}\right)$

$r=\sqrt{8+8}$

$r=4$

$\theta=\tan^{-1}\left(\frac{-\sqrt{8}}{\sqrt{8}}\right)$

$\theta=\frac{7\pi}{4}$

$\left(4,\frac{7\pi}{4}\right)$

19. $\left(\sqrt{6},\sqrt{2}\right)$

$r=\sqrt{6+2}$

$r=2\sqrt{2}$

$\theta=\tan^{-1}\left(\sqrt{\frac{1}{3}}\right)$

$\theta=\tan^{-1}\frac{\sqrt{3}}{3}$

$\theta=\frac{\pi}{6}$

$\left(2\sqrt{2},\frac{\pi}{6}\right)$

$\left(\frac{\sqrt{2}}{2},\frac{1}{2}\right)\to\frac{\pi}{6}$

20. $\left(-\sqrt{3},0\right)$

$(-\sqrt{3},\pi)$

For problems 21-24, convert the rectangular equation to polar form.

21. $y = x$

$r\sin\theta = r\cos\theta$

$\sin\theta = \cos\theta$

$1 = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = 1$

22. $y = 2x^2$

$r\sin\theta = 2(r\cos\theta)^2$

$r\sin\theta = 2r^2\cos^2\theta$

$r = \frac{\sin\theta}{2\cos^2\theta}$

23. $x = 8$

$r\cos\theta = 8$

$r = \frac{8}{\cos\theta}$

$r = 8\sec\theta$

24. $x^2 - y^2 = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$

For problems 25-30, convert the polar equation to rectangular form.

25. $r = 5$

$\sqrt{x^2+y^2} = 5$

$x^2 + y^2 = 25$

26. $\theta = \frac{5\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{5\pi}{6}$

$\tan\theta = \tan\frac{5\pi}{6}$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\tan\theta = -\frac{\sqrt{3}}{3}$

27. $r = 3\cos\theta$

$r = 3\left(\frac{x}{r}\right)$

$r^2 = 3x$

$x^2 + y^2 = 3x$

$x^2 + 3x + y^2 = 0$

28. $r^2 = \sin 2\theta$

$x^2 + y^2 = 2\sin\theta\cos\theta$

$x^2 + y^2 = 2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$

$x^2 + y^2 = \frac{2xy}{r^2}$

$(x^2+y^2)^2 = 2xy$

29. $r = \frac{4}{1+2\sin\theta}$

$r = \frac{4}{1+2\left(\frac{y}{r}\right)}$ $\frac{r}{r}$

$r = \frac{4r}{r+2y}$

$1 = \frac{4}{\sqrt{x^2+y^2}+2y}$

$\sqrt{x^2+y^2} + 2y = 4$

30. $\csc\theta = 2$

$\frac{r}{y} = 2$

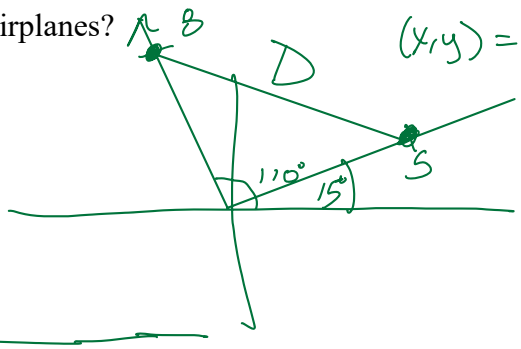
$r = 2y$

$\sqrt{x^2+y^2} = 2y$

31. A radar detects two airplanes at the same altitude. Their polar coordinates are $\left(8 \text{ miles}, 110^\circ\right)$ and $\left(5 \text{ miles}, 15^\circ\right)$. How far apart are the airplanes?

$(x,y) = (5\cos 15^\circ, 5\sin 15^\circ)$

$(x,y) = (8\cos 110^\circ, 8\sin 110^\circ)$



$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(8\cos 110^\circ - 5\cos 15^\circ)^2 + (8\sin 110^\circ - 5\sin 15^\circ)^2}$ ← exact answer w/o calculator

≈ 9.796 or 9.797 miles