Name KEY Date________________ Period______

Worksheet 7.2—Polar Equations

Show all work on a separate sheet of paper. **No Calculator** unless otherwise specified.

Multiple Choice

C

1. Which of the following gives the number of petals of the rose curve $r = 6\cos \underset{}{\overset{=n}{2}}\theta$?
(A) 1 (B) 2 (C) 4 (D) 6 (E) 12

even, so we see $2(\overset{n}{2}) = 4$ petals (from 0 to 2π)

A

2. Which of the following describes the symmetry of the rose graph of $r = 4\cos 3\theta$?
(A) x-axis only (B) y-axis only (C) origin only (D) all three (E) none

cosine, so x-axis symm

B

3. Which of the following is a maximum radius value for $r = 2 - 3\cos\theta$?
(A) 6 (B) 5 (C) 3 (D) 2 (E) 1

max radius is $2+3=5$ units

B

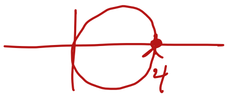
4. Which of the following is the number of petals of the rose curve $r = 8\sin \overset{=n}{5}\theta$?
(A) 1 (B) 5 (C) 8 (D) 10 (E) 16

5 is odd, so we see 5 petals (from 0 to π)

Short Answer

For questions 5 – 10, sketch the graph of each of the following polar equations by picking and plotting points. How many radians does it take to complete one closed cycle? Verify on your calculator.

5. $r = 4\cos\theta$

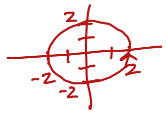


$\theta \in [0, \pi]$

off-set circle

diam = 2

6. $r = 2$

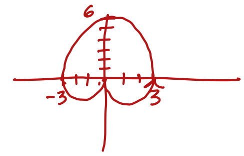


$\theta \in [0, 2\pi]$

circle

rad = 2

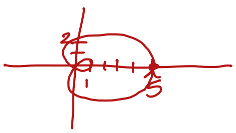
7. $r = 3 + 3\sin\theta$



cardioid

$\theta \in [0, 2\pi]$

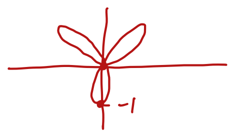
8. $r = 2 + 3\cos\theta$



Limaçon

$\theta \in [0, 2\pi]$

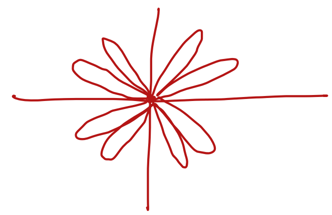
9. $r = \sin 3\theta$



rose curve

$\theta \in [0, \pi]$

10. $r = \sin 4\theta$



rose curve

$\theta \in [0, 2\pi]$

For questions 11 – 15, determine if the polar equation has any symmetry with respect to the polar axis (*x*-axis), the pole (orign), and/or the line $\theta = \frac{\pi}{2}$ (the *y*-axis).

11. $r = 3 - 2\sin\theta$

y-axis

12. $r = 5 + 7\cos\theta$

x-axis

13. $r = 2\sec\theta$

$r = \frac{2}{\cos\theta}$

(vertical line @ x = 2)

x-axis

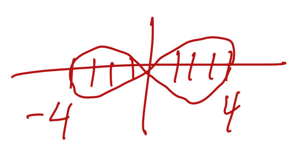
14. $r = 3\sin 3\theta$

y-axis

15. $r^2 = 16\cos\theta$

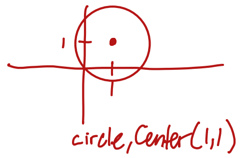
lemniscate (infinity symbol)

x-axis



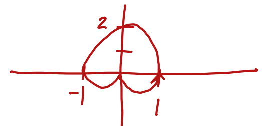
For questions 16 – 24, use your calculator to sketch the graph of each of the following polar equations in an appropriate window. Set your calculator to radian mode and your $\theta[0, 4\pi]$. Transfer the sketch to your paper.

16. $r = 2\cos\theta + 2\sin\theta$



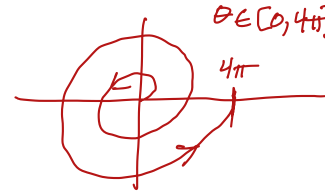
17. $r = \sin\theta - 1$

$r = -1 + |\sin\theta$



Cardioid

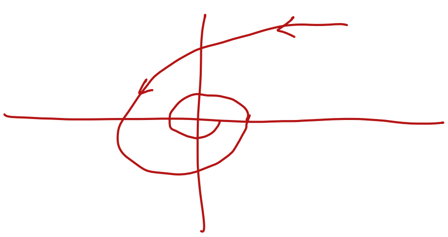
18. $r = \theta,\ \theta \ge 0$ (spiral)



19. $\theta = \dfrac{1}{r},\ \theta > 0$ (hyperbolic spiral)

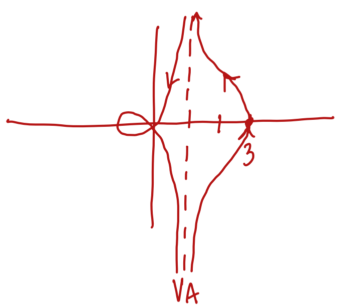
$r\theta = 1$

$r = \frac{1}{\theta}$

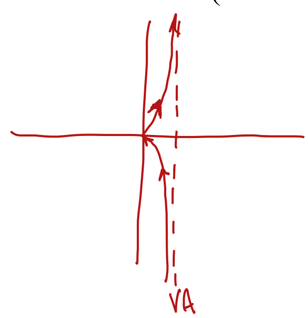


20. $r = 3 + \sec\theta$ (conchoid)

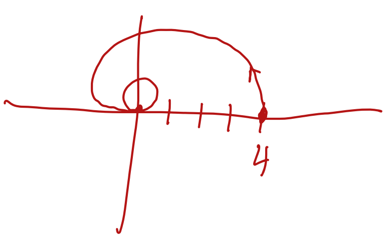
$r = 3 + \frac{1}{\cos\theta}$



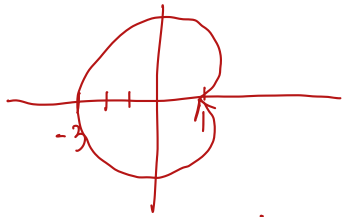
21. $r = \sin\theta\tan\theta$ (cissoid)



22. $r = \dfrac{4\sin\theta}{\theta}$ (cochleoid)

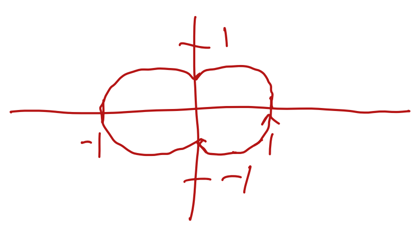


23. $r = 1 + 2\sin\left(\dfrac{\theta}{2}\right)$ (nephroid)



(off-set cardioid)

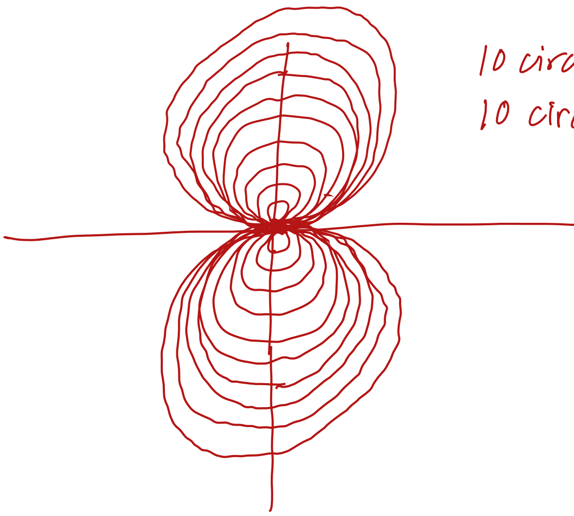
24. $r = \sqrt{1 - 0.8\sin^2\theta}$ (hippopede)



Peanut?

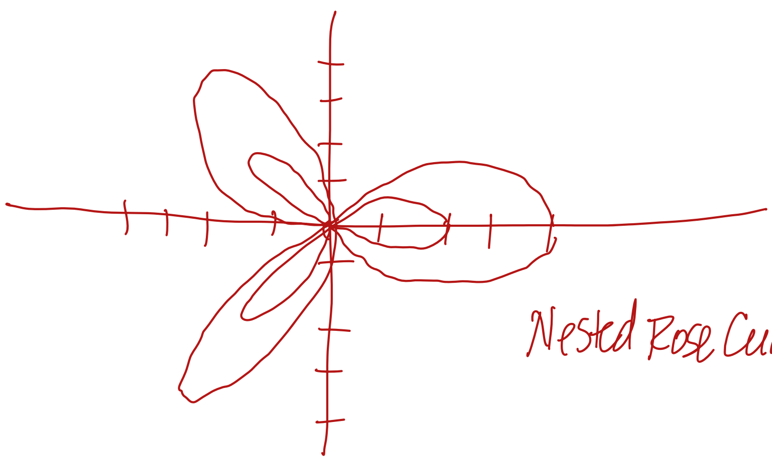
For questions 25 – 27, graph each of the following on your calculator in the with the given settings, then sketch on your paper.

25. $r = \theta \sin\theta$, $\theta : [-10\pi, 10\pi]$, $x : [-40, 40]$, $y : [-30, 30]$



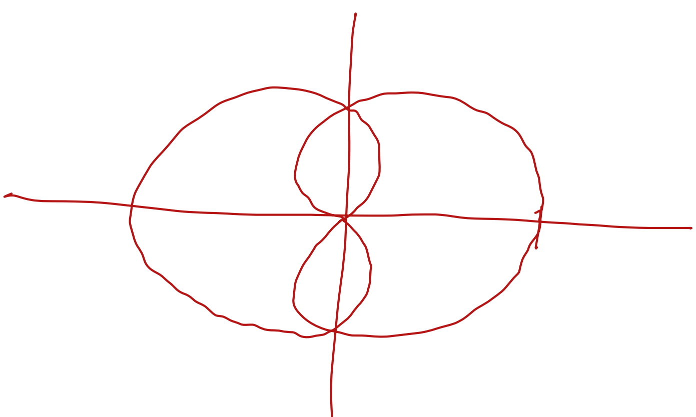
10 circles up
10 circles down

26. $r = 1 + 3\cos(3\theta)$, $\theta : [0, 2\pi]$, $x : [-5, 5]$, $y : [-4, 4]$



Nested Rose Curve

27. $r = \sin\left(\frac{\theta}{2}\right)$, $\theta : [-2\pi, 2\pi]$, $x : [-1.25, 1.25]$, $y : [-1, 1]$



y-axis reflected cardioids

$x = r\cos\theta$, $y = r\sin\theta$

$x^2 + y^2 = r^2$

For questions 28 – 29, sketch the graph of the rectangular equation by first converting it into an equivalent polar equation. → solve for r

28. $(x^2 + y^2)^3 = 4x^2y^2$

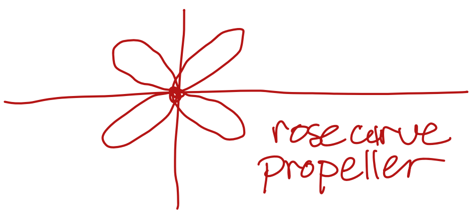
$(r^2)^3 = 4(r\cos\theta)^2(r\sin\theta)^2$

$r^6 = 4r^2\cos^2\theta\, r^2\sin^2\theta$

$r^6 = 4r^4\cos^2\theta\sin^2\theta$

$r^2 = 4\cos^2\theta\sin^2\theta$

$r = \pm 2\cos\theta\sin\theta$



rose curve

propeller

29. $x^2 + y^2 = (x^2 + y^2 - x)^2$

$r^2 = (r^2 - r\cos\theta)^2$

$r^2 = r^4 - 2r^3\cos\theta + r^2\cos^2\theta$

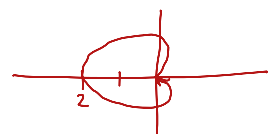
$r^2 = r^2(r^2 - 2r\cos\theta + \cos^2\theta)$

$1 = r^2 - 2r\cos\theta + \cos^2\theta$

$1 = (r - \cos\theta)^2$

$r - \cos\theta = \pm 1$

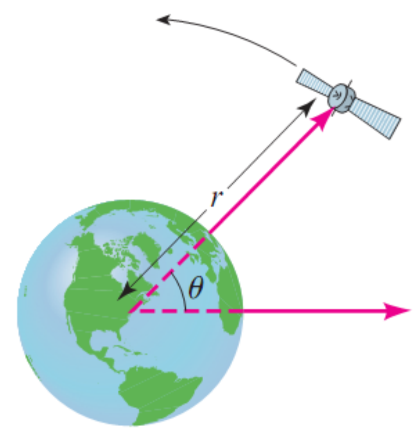
$r = 1 - \cos\theta$ & $r = -1 - \cos\theta$



cardioid

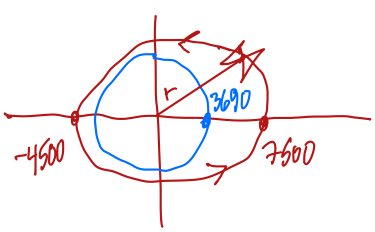
Application:

30. A satellite orbits the earth. It's orbit is modeled by the equation $r = \dfrac{22500}{4 - \cos\theta}$, where r is the distance in miles between the satellite and the center of the earth and θ is the angle shown in the figure below.



(a) On the same viewing screen on your calculator (you must decide on an appropriate viewing window), graph the circle $r = 3690$ (to represent planet earth) and the equation of the satellite's orbit. Describe the motion of the satellite as θ increases from 0 to 2π. You can use your "trace" feature for this.

window: $x \in [-6000, 10000]$, $y \in [-6000, 6000]$



the satellite moves counterclockwise around Earth, closest at 4500 miles from Earth's center, farthest at 7500 miles

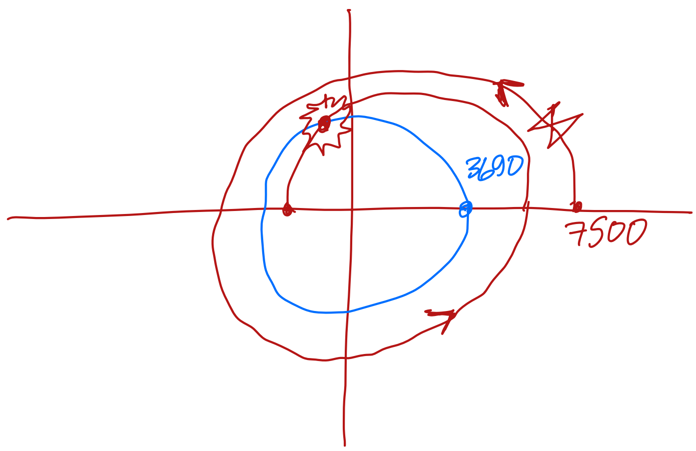
(b) For what angle θ is the satellite closest to the earth? Find the height of the satellite above the earth's surface for this value of θ.

Closest when $\theta = \pi$. At this point, it is 4500 miles from Earth's center, or $4500 - 3690 = 810$ miles above Earth's surface.

(c) The orbit described above is stable because the satellite traverses the same path over and over as θ increases. Suppose that a meteor strikes the satellite and changes its orbit to $r = \dfrac{22500\left(1-\dfrac{\theta}{40}\right)}{4-\cos\theta}$.

OH, NO!

On the same viewing screen, graph the equation representing earth and the satellites new orbit. Describe the new motion of the satellite as θ increases from 0 to 3π (change θmax to 3π).



the satellite still orbits counterclockwise, but now in a decreasing spiral until it strikes Earth's surface in Quadrant II

(d) Use the "trace" feature on your calculator to find the value of θ at the moment the satellite crashes into earth.

The satellite strikes the Earth at approximately $\theta = 8.770$ radians. oops! (stupid meteor!)