Name KEY Date Period

Worksheet 7.3—Plane Curves & Parametric Equations

Show all work on a separate sheet of paper. **No Calculator** unless otherwise specified.

Multiple Choice

A

1. Which of the following points corresponds to $t=-1$ in the parameterization $x=t^2-4$, $y=t+\frac{1}{t}$?

(A) $(-3,-2)$ (B) $(-3,0)$ (C) $(-5,-2)$ (D) $(-5,0)$ (E) $(3,2)$

$(x(-1), y(-1))$
$(1-4, -1-1)$
$(-3,-2)$

A

2. Which of the following values of t produces the same point as $t=\frac{2\pi}{3}$ in the parametrization $x=2\cos t$, $y=2\sin t$?

(A) $t=-\frac{4\pi}{3}$ (B) $t=-\frac{2\pi}{3}$ (C) $t=-\frac{\pi}{3}$ (D) $t=\frac{4\pi}{3}$ (E) $t=\frac{7\pi}{3}$

$(x(\frac{2\pi}{3}), y(\frac{2\pi}{3}))$
$(2\cos\frac{2\pi}{3}, 2\sin\frac{2\pi}{3})$
$(-1, \sqrt{3})$

$\frac{2\pi}{3} - \frac{4\pi}{3}$ coterminal with

D

3. A rock is thrown straight up from level ground with its position above ground at any time $t\geq 0$ given by $x=5$, $y=-16t^2+80t+7$. At what time will the rock be 91 feet above ground?

(A) 1.5 sec (B) 2.5 sec (C) 3.5 sec (D) 1.5 sec and 3.5 (E) NEVER

$y=91$
$91=-16t^2+80t+7$
$-16t^2+80t-84=0$
$-4(4t^2-20t+21)=0$
$-4(2t-3)(2t-7)=0$

$t=\frac{3}{2}$ sec up, $\frac{7}{2}$ sec down

C

4. Which of the following describes the graph of the parametric equations $x=1-t$, $y=3t+2$, $t\geq 0$?

(A) a straight line (B) a line segment (C) a ray (D) a parabola (E) a circle

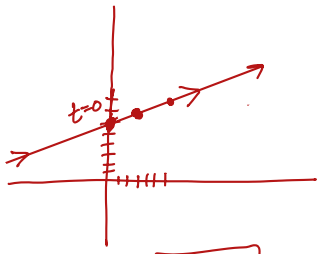
$x=1-t$
$t=1-x \rightarrow y=3(1-x)+2$
$y=3-3x+2$
$y=-3x+5$
a line starting at $(x(0), y(0))=(1,2)$ and going forever in one direction

Short Answer

*calculator OK

For each of the following 5 – 10, a pair of parametric equations is given. (a) sketch the curve represented by the parametric equations showing the path and direction, then (b) find a rectangular-coordinate equation for the curve by eliminating the parameter.

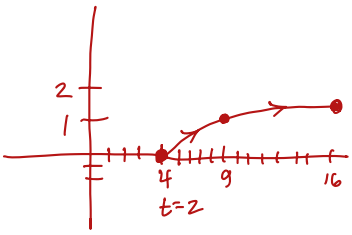
5. $x = 2t,\ y = t + 6$ $t \in (-\infty, \infty)$



$t = \frac{x}{2} \rightarrow$ $y = \frac{x}{2} + 6$

Line

6. $x = t^2,\ y = t - 2,\ 2 \le t \le 4$

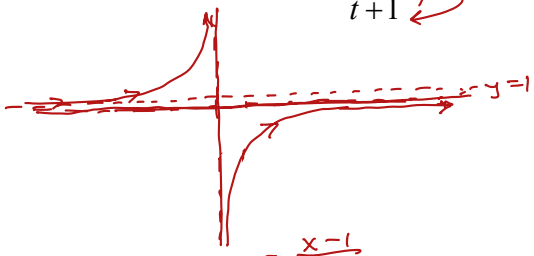


$t = y + 2 \rightarrow x = (y+2)^2$

$y + 2 = \pm\sqrt{x},\ x \ge 4$

$y = \sqrt{x} - 2$

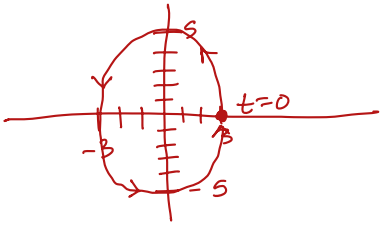
7. $x = t + 1,\ y = \frac{t}{t+1}$ $t \ne -1$



$t = x - 1 \rightarrow y = \frac{x-1}{x-1+1}$

$y = \frac{x-1}{x}$ $x \ne 0$ VA HA $ey = 1$

8. $x = 3\cos t,\ y = 5\sin t,\ 0 \le t \le 2\pi$



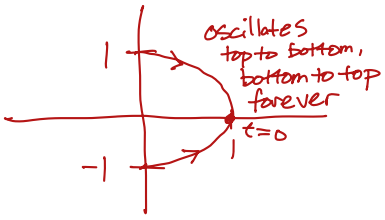
ellipse

$\cos t = \frac{x}{3},\ \sin t = \frac{y}{5}$

$\cos^2 t + \sin^2 t = 1$

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

9. $x = \sin^2 t,\ y = \cos t$ $t \in (-\infty, \infty)$



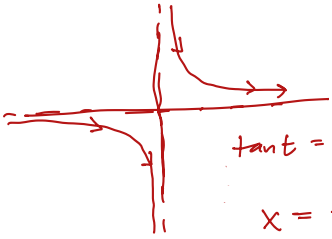
$\sin^2 t + \cos^2 t = 1$

$x + y^2 = 1$

$y^2 = 1 - x$

$y = \pm\sqrt{1-x}$

10. $x = \tan t,\ y = \cot t,\ 0 < t < \frac{\pi}{2}$



$\tan t = \frac{1}{\cot t}$

$x = \frac{1}{y}$

$y = \frac{1}{x}$ $x \ne 0$

For problems 11 & 12, find parametric equations for the line with the given properties.

11. slope of $\frac{1}{2}$, passing through $(4, -1)$

$y = \frac{1}{2}x + b$

@ $(4,-1)$: $-1 = \frac{1}{2}(4) + b$

$-1 = 2 + b$

$b = -3$

You can always Let $x = t$

$y = \frac{1}{2}x - 3$

Let $x = t,\ y = \frac{1}{2}t - 3$

$(4,-1)$ when $t = 4$

12. passing through $(6,9)$ and $(9,12)$

$m = \frac{12-9}{9-6}$

$m = \frac{3}{3}$

$m = 1$

$y = 1x + b$

@ $(6,9)$: $9 = 6 + b$

$b = 3$

$y = x + 3$

Let $x = 2t,\ y = 2t + 3$

$(6,9)$ when $t = 3$

13. Find parametric equations for the circle $x^2 + y^2 = r^2$, centered at the origin with radius of r.

Let $x = t$, $y = \pm\sqrt{r^2 - t^2}$ for some r

14. (Calculator permitted) Suppose a gun fires a bullet into the air from a height of 5 feet with an initial speed of 2048 ft/s at an angle of 30° to the horizontal.

$30^\circ = \frac{\pi}{6}$ rads

$y(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$

$x(t) = v_0 t$

(a) After how many seconds will the bullet hit the ground?

initial velocities:

$x(0) = 2048\cos\frac{\pi}{6} = 2048\left(\frac{\sqrt{3}}{2}\right) = 1024(\sqrt{3})$

$y(0) = 2048\sin\frac{\pi}{6} = 2048\left(\frac{1}{2}\right) = 1024$

So, $y(t) = -\frac{1}{2}(32)t^2 + 1024t + 5$

$y(t) = -16t^2 + 1024t + 5 = 0$

$t = 64.004$ sec $= A$ (store as A)

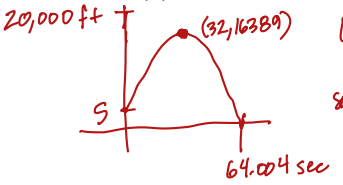
(b) How far from the gun will the bullet hit the ground?

$x(t) = v_0 t$

$x(A) = 1024\sqrt{3}(64.004\ldots)$ (A)

$= 113520.341$ ft

(c) What is the maximum height attained by the bullet?

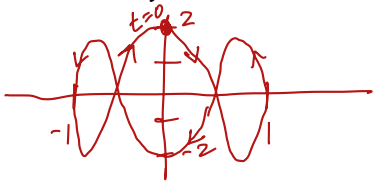


2nd trace maximum

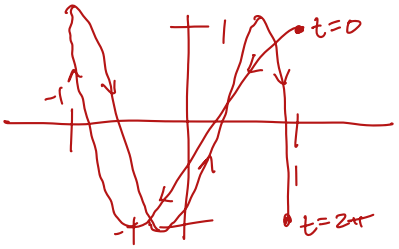
So, Max height is 16,389 ft.

For 15 – 17, using your graphing calculator, sketch the curve represented by the following parametric equations. Transfer the graph to your paper showing the path and direction.

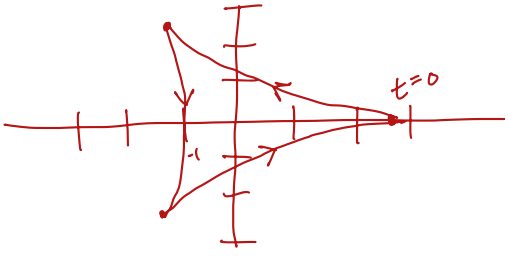
15. $x = \sin t$, $y = 2\cos 3t$ $t \in (-\infty, \infty)$



16. $\sin(\cos t)$, $y = \cos(t^{3/2})$, $0 \le t \le 2\pi$



17. $x = 2\cos t + \cos 2t$, $y = 2\sin t - \sin 2t$, $t \in (-\infty, \infty)$



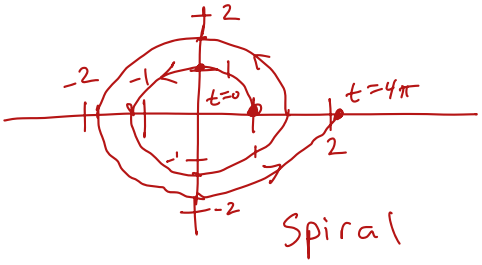
For18 & 19, a polar equation is given. (a) express the polar equation in parametric form, then (b) use your graphing calculator to graph the parametric equations. Transfer the graph to your paper showing the direction and path.

18. $r = 2^{\theta/12}$, $0 \le t \le 4\pi$

$t=\theta$

$x=r\cos\theta \quad y=r\sin\theta$

$x=2^{t/12}\cos t \quad y=2^{t/12}\sin t$



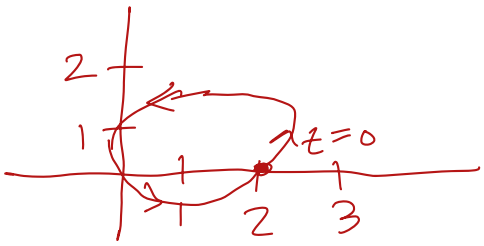
Spiral

19. $r = \sin\theta + 2\cos\theta$, $t \in (-\infty, \infty)$

$\theta = t$

$x=r\cos\theta \quad y=r\sin\theta$

$x=(\sin t+2\cos t)\cos t \quad y=(\sin t+2\cos t)\sin t$



ellipse

20. (Calculator permitted) The curves *A, B, C,* and *D* are defined parametrically as follows, where the parameter *t* takes on all real values unless stated otherwise:

$$A:\ x = t,\ y = t^2$$
$$B:\ x = \sqrt{t},\ y = t,\ t \ge 0$$
$$C:\ x = \sin t,\ y = \sin^2 t$$
$$D:\ x = 3^t,\ y = 3^{2t}$$

(a) Show that the points on all fourn of these curves satisfy the same rectangular coordinate equation.

(A) $x=t \to y=x^2$ (B) $x=\sqrt{t}$ $t=x^2 \to y=x^2$ (C) $x=\sin t \to y=(\sin t)^2$ $y=x^2$ (D) $x=3^t \to y=(3^t)^2$ $y=x^2$

(b) Draw the graph of each curve and explain how the curves differ from each other. Be sure to address path, speed, and direction.

Slower than (A) restricted path

Smaller path back & forth

restricted path faster than (A)

21. (Calculator Permitted) Archimedes hits a baseball when it is 4 feet above the ground with an intitial velocity of 120 feet per second. The ball leaves the bat at a 30° angle with the horizontal and heads toward a 30-foot fence 350 feet from home plate.

(a) Write a set of parametric equations representing the height of the ball and its distance from home plate.

initial velocity: $x(0)=120\cos\frac{\pi}{6}$ $y(0)=120\sin\frac{\pi}{6}$

$x(0)=60\sqrt{3}$ $y(0)=60$

$y(t)=-16t^2+60t+4$, $x(t)=60\sqrt{3}t$, $t\geq 0$

(b) If so, by how much does the baseball clear the fence? If not, could the ball be caught? Justify.

$x(t)=350$

$60\sqrt{3}t=350$

$t=\frac{350}{60\sqrt{3}}$

$t\approx 3.367\ldots = A$ (store as A)

$y(A)=24.591$

the fence is 30ft tall, so the ball will not clear the fence (5.408ft short), and will likely not be caught unless the player can climb the wall to reach the ball 24.591 ft in the air!

(c) suppose that the moment Archimedes hits the ball, there is a 5-foot per second wind gust. Assuming that this horizontal wind gusts acts in the horizontal direction out with the ball, does the ball clear the fence? If so, by how much? If not, could the ball be caught? Justify.

$x(t)=(60\sqrt{3}+5)t=350$

$t=\frac{350}{60\sqrt{3}+5}$

$t\approx 3.213$ sec $= B$

$y(B)=31.594$

with the wind, the ball clears the fence with 1.594 ft to spare.

Archimedes hits a home run!!