Name KEY Date____________________ Period______

Worksheet 7.4—Vectors

Show all work on a separate sheet of paper. **No Calculator** unless otherwise specified.

Multiple Choice

D

1. Which of the following is the magnitude of the vector $\langle 2,-1\rangle$? $=\vec{v}$

(A) 1 (B) $\sqrt{3}$ (C) $\dfrac{\sqrt{5}}{5}$ (D) $\sqrt{5}$ (E) 5

$\|\vec{v}\| = \sqrt{2^2+(-1)^2}$
$= \sqrt{4+1}$
$= \sqrt{5}$

E

2. Let $\vec{u}=\langle -2,3\rangle$ and $\vec{v}=\langle 4,-1\rangle$. Which fo the following is equal to $\vec{u}-\vec{v}$?

(A) $\langle 6,-4\rangle$ (B) $\langle 2,2\rangle$ (C) $\langle -2,2\rangle$ (D) $\langle -6,2\rangle$ (E) $\langle -6,4\rangle$

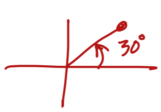
$\vec{u}-\vec{v} = \langle -2-4, 3-(-1)\rangle$
$= \langle -6,4\rangle$

C

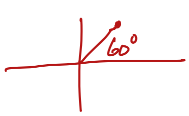
3. Which of the following represents the vector $\vec{v}$ with a magnitude of 3 and a direction of 150°?

(A) $\left\langle \dfrac{3\sqrt{3}}{2},\dfrac{3}{2}\right\rangle$ (B) $\left\langle \dfrac{3}{2},\dfrac{3\sqrt{3}}{2}\right\rangle$ (C) $\left\langle -\dfrac{3\sqrt{3}}{2},\dfrac{3}{2}\right\rangle$ (D) $\left\langle -\dfrac{3}{2},\dfrac{3\sqrt{3}}{2}\right\rangle$ (E) $\left\langle -\dfrac{3}{2},-\dfrac{3\sqrt{3}}{2}\right\rangle$

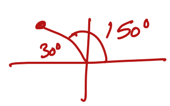
$\|\vec{v}\| = \sqrt{\frac{27}{4}+\frac{9}{4}}$
$= \sqrt{9}$
$= 3$



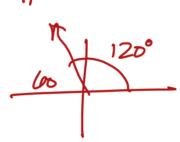
$\|\vec{v}\| = 3$



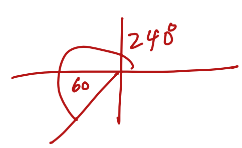
$\|\vec{v}\| = 3$



$\|\vec{v}\| = 3$



$\|\vec{v}\| = 3$



C 4. Which of the following is a unit vector in the direction of $\vec{v} = -\vec{i} + 3\vec{j}$?

(A) $-\frac{1}{10}\vec{i} + \frac{3}{10}\vec{j}$ (B) $\frac{1}{10}\vec{i} - \frac{3}{10}\vec{j}$ (C) $-\frac{1}{\sqrt{10}}\vec{i} + \frac{3}{\sqrt{10}}\vec{j}$ (D) $\frac{1}{\sqrt{10}}\vec{i} - \frac{3}{\sqrt{10}}\vec{j}$ (E) $-\frac{1}{\sqrt{8}}\vec{i} + \frac{3}{\sqrt{8}}\vec{j}$

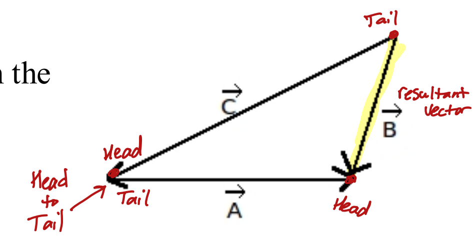
$\|\vec{v}\| = \sqrt{1+9}$
$= \sqrt{10}$
$\vec{u} = -\frac{1}{\sqrt{10}}\vec{i} + \frac{3}{\sqrt{10}}j$

D 5. Which of the following is a vector quantity?
(A) Energy (B) Power (C) Time (D) Force (E) Mass

A 6. Which of the following is a scalar quantity?
(A) Area (B) Kinetic Energy (C) Weight of an object (D) Wind velocity (E) Acceleration

C 7. Which of the following gives the correct relation among the vectors in the diagram at right.

(A) $\vec{A} + \vec{B} = \vec{C}$ (B) $\vec{B} + \vec{C} = \vec{A}$ (C) $\vec{C} + \vec{A} = \vec{B}$ (D) $\vec{A} + \vec{B} + \vec{C} = 0$

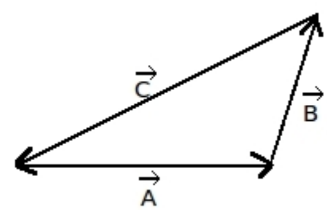


$\vec{C} + \vec{A} = \vec{B}$

D 8. Which of the following gives the correct relation among the vectors in the diagram at right.

(A) $\vec{A} + \vec{B} = \vec{C}$ (B) $\vec{B} + \vec{C} = \vec{A}$ (C) $\vec{C} + \vec{A} = \vec{B}$ (D) $\vec{A} + \vec{B} + \vec{C} = 0$

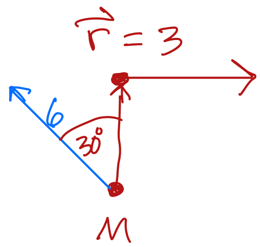
Back where started

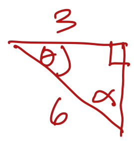


B

9. A straight river flows west to east at a speed of 3 meters per minute. A man on the south bank of the river, capable of swimming at 6 meters per minute in still water, wants to swim across to the point directly opposite the bank. He should swim in a direction of

(A) due north (B) 30° west of north (C) 30° east of north (D) 60° east of north (E) south





$\cos\theta = \frac{1}{2}$

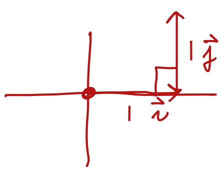
$\theta = 60^\circ$

$\alpha = 30^\circ$

D

10. The angle between $\vec{i}+\vec{j}$ and $\vec{i}$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{5\pi}{6}$



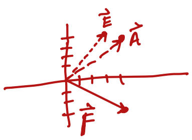
E

11. Consider a vector $\overline{F} = 4\vec{i} - 3\vec{j}$. The vector which is perpendicular to $\overline{F}$ is

90°

(A) $4\vec{i}+3\vec{j}$ (B) $-4\vec{i}+3\vec{j}$ (C) $-3\vec{i}+4\vec{j}$ (D) $3\vec{i}-4\vec{j}$ (E) $3\vec{i}+4\vec{j}$

180°



$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

73.739°

* property

$\tan^{-1}\left(\frac{A}{B}\right) + \tan^{-1}\left(\frac{B}{A}\right) = 90^\circ$

$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

90°

B

12. The unit vector along $\vec{i}+\vec{j}$ is

(A) $\vec{i}+\vec{j}$ (B) $\frac{\sqrt{2}}{2}\vec{i}+\frac{\sqrt{2}}{2}\vec{j}$ (C) $\frac{1}{2}\vec{i}+\frac{1}{2}\vec{j}$ (D) $\sqrt{2}\vec{i}+\sqrt{2}\vec{j}$ (E) none of these

$\|\vec{v}\| = \sqrt{1+1}$

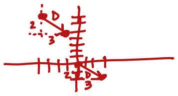
$= \sqrt{2}$

$\vec{u} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$

$= \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$

Short Answer

13. Show that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent by showing that they represent the same vector: $A=(-4,7)$, $B=(-1,5)$, $C=(0,0)$, and $D=(3,-2)$



$D_{\vec{AB}} = \sqrt{(5-7)^2+(-1--4)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$D_{\vec{CD}} = \sqrt{(-2)^2+(3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

same slope

$-\frac{2}{3}$

So, same angle

So, same vector (since same magnitude)

14. Given the points, $A=(-2,2)$, $B=(3,4)$, $C=(-2,5)$, and $D=(2,-8)$, find the indicated vector in $\vec{u}=\langle a,b\rangle$ chevron form, then find the magnitude, $|\vec{u}|$ of the vector.

(a) $\overline{BC}$

$\vec{u} = \vec{BC} = (-2-3)\vec{i} + (5-4)\vec{j}$

$= \langle -5, 1\rangle$

$\|\vec{u}\| = \sqrt{25+1}$

$= \sqrt{26}$

(b) $4\overline{AC}$

$\vec{u} = 4\langle -2--2, 5-2\rangle$

$\vec{u} = 4\langle 0,3\rangle$

$\vec{u} = \langle 0,12\rangle$

$\|\vec{u}\| = 12$

(c) $3\overline{BD} - 2\overline{AC}$

$\vec{u} = 3\langle 2-3, -8-4\rangle - 2\langle -2--2, 5-2\rangle$

$\vec{u} = 3\langle -1,-12\rangle - 2\langle 0,3\rangle$

$\vec{u} = \langle -3,-36\rangle + \langle 0,-6\rangle$

$\vec{u} = \langle -3,-42\rangle$

$\|\vec{u}\| = \sqrt{3^2+42^2}$

$= \sqrt{1773}$

≈ 42.107

15. Given the vectors $\vec{u}=\langle 1,3\rangle$, $\vec{v}=\langle 2,4\rangle$, and $\vec{w}=\langle 2,-5\rangle$, find the following vectors in $\vec{i},\vec{j}$ form. Find the magnitude of each new vector.

(a) $\vec{v}+\vec{w} = \vec{u}$

$\vec{u} = (2+2)\vec{i} + (4-5)\vec{j}$

$= 4\vec{i} - \vec{j}$

$\|\vec{u}\| = \sqrt{16+1}$

$= \sqrt{17}$

≈ 4.123

(b) $-\vec{w}+\frac{1}{2}\vec{v} = \vec{u}$

$\vec{u} = (-2+1)\vec{i} + (+5+2)\vec{j}$

$= -\vec{i} + 7\vec{j}$

$\|\vec{u}\| = \sqrt{1+49}$

$= \sqrt{50}$

≈ 7.071

(c) $2\vec{u} - 3v = \vec{u}$

$\vec{u} = (2-6)\vec{i} + (6-12)\vec{j}$

$= -4\vec{i} - 6\vec{j}$

$\|\vec{u}\| = \sqrt{16+36}$

$= \sqrt{50}$

≈ 7.071

16. Find a unit vector in the direction of $-2\vec{i}+3\vec{j}$ $=\vec{r}$. Write your answer in **both** chevron and $\vec{i},\vec{j}$ form.

$\|\vec{r}\|=\sqrt{4+9}$
$=\sqrt{13}$

$\vec{u}=-\frac{2}{\sqrt{13}}\vec{i}+\frac{3}{\sqrt{13}}\vec{j}$

$=\left\langle -\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right\rangle$

17. Find the horizontal and vertical components of the vector with the given length and direction, then write the vector in $\vec{i},\vec{j}$ form.

$\cos 125^\circ=-0.573=A$
$\sin 125^\circ=0.819=B$

(a) $|\vec{v}|=50$, $\theta=\frac{7\pi}{6}$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\vec{v}=50\cos\frac{7\pi}{6}\vec{i}+50\sin\frac{7\pi}{6}\vec{j}$
$=50\left(-\frac{\sqrt{3}}{2}\right)\vec{i}+50\left(-\frac{1}{2}\right)\vec{j}$
$=-25\sqrt{3}\,\vec{i}-25\vec{j}$

(b) $|\vec{u}|=13$, $\theta=330^\circ$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

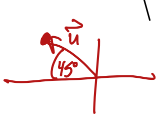
$\vec{u}=13\cos 330^\circ\vec{i}+13\sin 330^\circ\vec{j}$
$=13\left(\frac{\sqrt{3}}{2}\right)\vec{i}+13\left(-\frac{1}{2}\right)\vec{j}$
$=\frac{13\sqrt{3}}{2}\vec{i}-\frac{13}{2}\vec{j}$

(c) (Calculator) $|\vec{r}|=900$, $\theta=125^\circ$

$\vec{r}=900\cos 125\vec{i}+900\sin 125\vec{j}$
$\vec{r}=900A\vec{i}+900B\vec{j}$
$\vec{r}=-516.218\vec{i}+737.236\vec{j}$

18. Find the magnitude and direction (in degrees) of the following vectors.

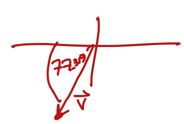
(a) $\vec{u}=\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$



$\|\vec{u}\|=\sqrt{\frac{1}{2}+\frac{1}{2}}$
$=\frac{1}{2}$

$\tan^{-1}|-1|=45^\circ$
$\theta=135^\circ$

(b) (Calculator) $\vec{v}=-9\vec{i}-40\vec{j}$

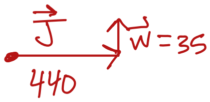


$\|\vec{v}\|=\sqrt{81+1600}$
$=41$

$\theta_{ref}=\tan^{-1}\left|\frac{40}{9}\right|$
$=77.319$
$\theta=180+77.319$
$=257.319^\circ$

19. (Calculator) Corny the jet pilot heads his jet due east. The jet has a speed of 440 mph relative to the air. The wind is blowing due north with a speed of 35 mph.

(a) Express the velocity of the wind as a vector in component form.



$\vec{w} = 0\vec{i} + 35\vec{j}$

(b) Express the velocity of the jet relative to the air as a vector in component form.

$\vec{J} = 440\vec{i} + 0\vec{j}$

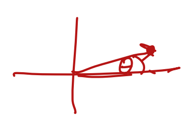
(c) Find the true velocity of the jet as a vector.

$$\vec{T} = \vec{w} + \vec{J}$$
$$= 440\vec{i} + 35\vec{j}$$

(d) Find the true speed and direction of the jet.

$$\|\vec{T}\| = \sqrt{440^2 + 35^2}$$
$$= \sqrt{194825}$$
$$= 441.389 \text{ mph}$$

$$\theta = \tan^{-1}\left(\frac{35}{440}\right)$$
$$= 38.500°$$



So, Corny is actually flying 38.500° North of East at 441.389 mph