Name___*KEY*_____ Date_____ Period_____

Worksheet 9.2—Conic Sections: Ellipses

Show all work. No calculator is permitted, unless explicitly stated.

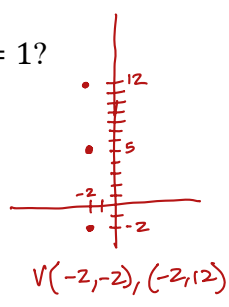### Multiple Choice

A 1. The line containing the foci is/are the

(A) Major Axis   (B) Minor Axis   (C) Vertices   (D) Covertices
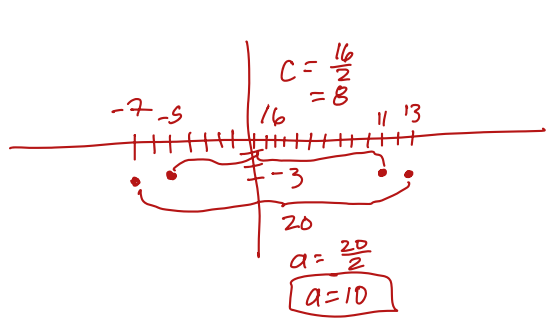
C 2. What are the vertices of the ellipse whose equation is $\frac{(x+2)^2}{9} + \frac{(y-5)^2}{49} = 1$?

(A) $(-5,5)$ & $(1,5)$ (B) $(-2,8)$ & $(-2,2)$
(C) $(-2,12)$ & $(-2,-2)$ (D) $(-9,5)$ & $(5,5)$

$V(-2,-2), (-2,12)$

_____ 3. What are the values of a and b for an ellipse with the following vertices and foci?
   Vertices: $(-7,-3)$ & $(13-3)$
   Foci: $(-5,-3)$ & $(11,-3)$

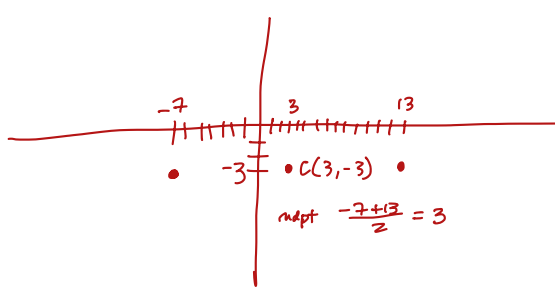(A) $a = 6$ & $b = 10$ (B) $a = 10$ & $b = 6$
(C) $a = 36$ & $b = 100$ (D) $a = 100$ & $b = 36$

$$a^2 - b^2 = c^2$$
$$100 - b^2 = 64$$
$$b^2 = 36$$
$$\boxed{b = 6}$$

$C = \frac{16}{2} = 8$

$a = \frac{20}{2}$
$\boxed{a = 10}$

D 4. What is the center of an ellipse whose vertices are at $(-7,-3)$ & $(13,-3)$ and whose foci are at $(-5,-3)$ & $(11,-3)$?

(A) $(10,6)$ (B) $(-3,3)$ (C) $(6,-6)$ (D) $(3,-3)$

$C(3,-3)$

mdpt $\frac{-7+13}{2} = 3$

_____C_____ 5. Find the foci of the ellipse whose equation is $25(x-4)^2 + 9(y+1)^2 = 225$

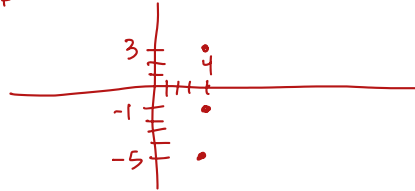(A) $(8,-1)$ & $(0,-1)$ (B) $(-1.8,-1)$ & $(9.8,-1)$

(C) $(4,3)$ & $(4,-5)$ (D) $(4,4.8)$ & $(4,-6.8)$

$$\frac{(x-4)^2}{9} + \frac{(y+1)^2}{25} = 1$$

Center $(4,-1)$

Mjr axis vert

$a=5, b=3$

$a^2-b^2=c^2$
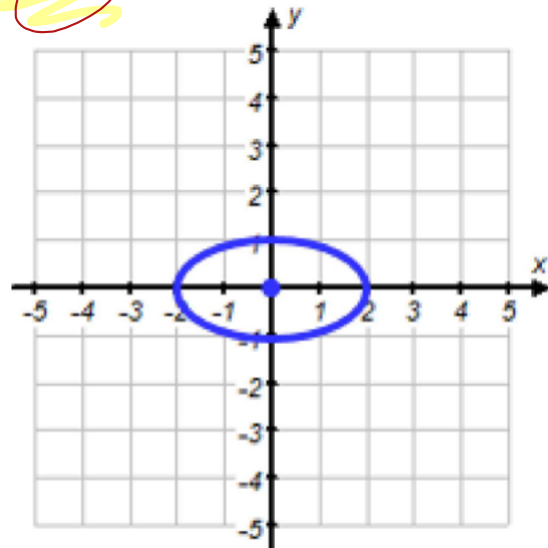
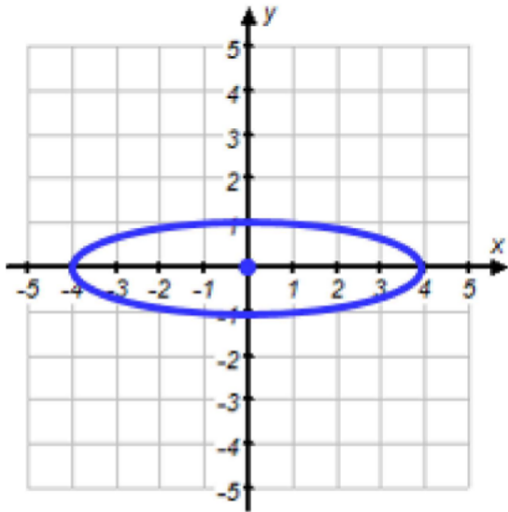$25-9=c^2$

$\boxed{c=4}$

Foci $(4,3), (4,-5)$

_____B_____ 6. Graph the ellipse whose standard equation is $\frac{x^2}{4} + \frac{y^2}{1} = 1$.
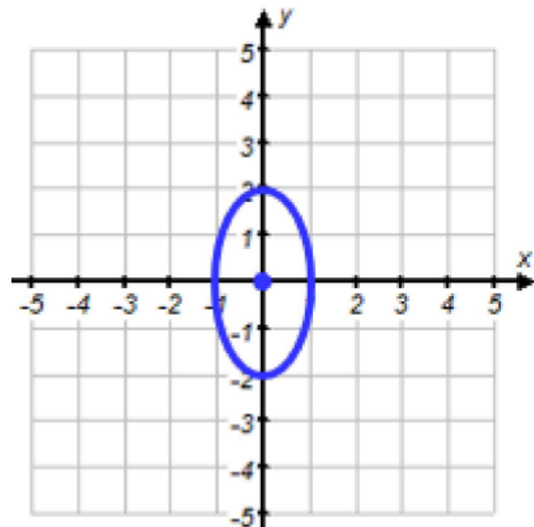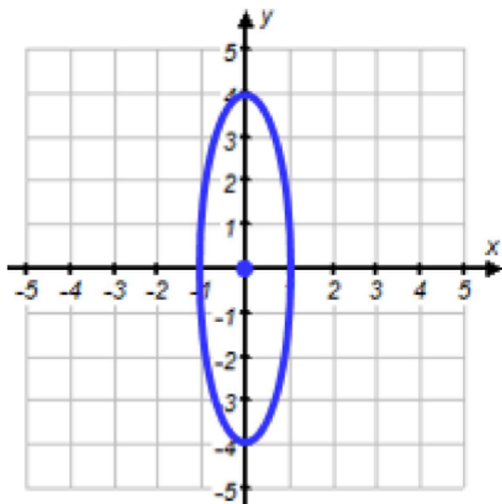
Center $(0,0)$

Horz mjr axis

$a=2$

(A)

(B)

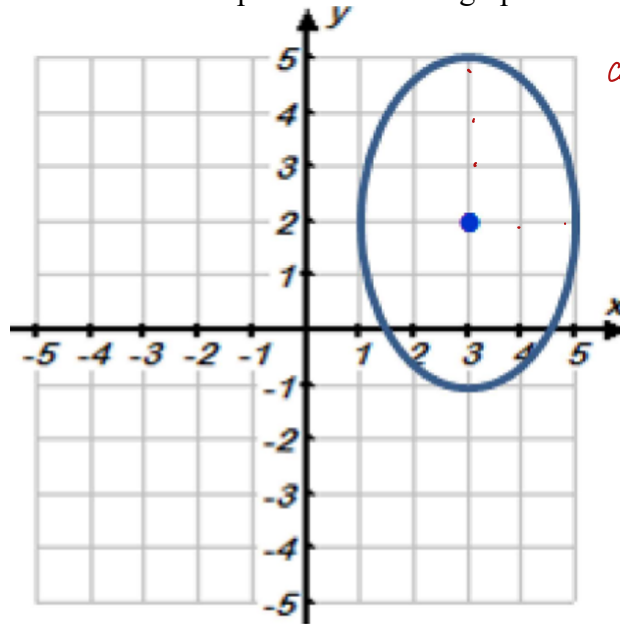(C)

(D)

A 7. Which equation best describes the ellipse shown in the graph below?

Center $(3,2)$
$a=3, b=2$
Vert mjr axis

$$\frac{(x-3)^2}{4} + \frac{(y-2)^2}{9} = 1$$

(A) $\frac{(x-3)^2}{4} + \frac{(y-2)^2}{9} = 1$

(B) $4(x-3)^2 + 9(y-2)^2 = 1$

(C) $\frac{(x+3)^2}{4} + \frac{(y+2)^2}{9} = 1$

(D) $4(x+3)^2 + 9(y+2)^2 = 1$

A 8. What are the coordinates of the center of the ellipse whose equation is
$4x^2 - 16x + y^2 + 6y = -21$?

(A) $(2,-3)$ (B) $(4,1)$ (C) $(2,1)$ (D) $(8,-3)$

$4(x^2 - 4x + 4) + y^2 + 6y + 9 = -21 + 16 + 9$

$4(x-2)^2 + (y+3)^2 = 4$

$c(2,-3)$

B 9. What are the coordinates of the vertices of the ellipse whose equation is
$x^2 - 2x + 4y^2 - 16y = -13$?

(A) (1,4) & (1,0)

(B) $(-1,2)$ & (3,2)

(C) (1,3) & (1,1)

(D) (1,2) & (5,2)

$x^2 - 2x + 1 + 4(y^2 - 4y + 4) = -13 + 1 + 16$

$(x-1)^2 + 4(y-2)^2 = 4$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{1} = 1$

$c(1,2)$, $a=2$ (Horz)

$V(1-2,2), (1+2,2)$

$V(-1,2)(3,2)$

C 10. A construction worker shined his flashlight on the floor at an angle which created an ellipse. A. coordinate grid was laid out on the floor such that the ellipse created was centered at the origin. The longest horizontal part of the ellipse created was 12 feet in length on the x-axis, and the shortest vertical length was 8 feet in length on the y-axis.
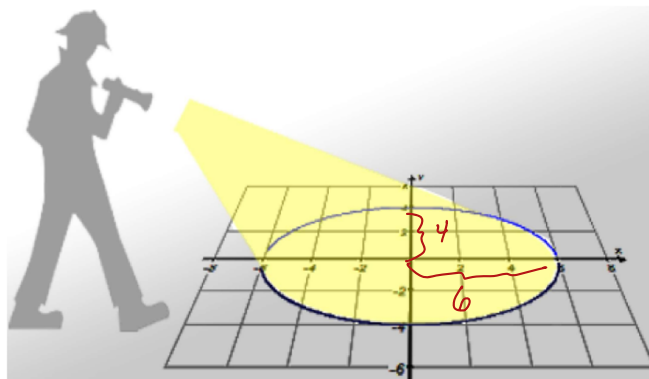
What would be an standard equation that describes the ellipse created by the flashlight on the floor?

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

(A) $\frac{x^2}{12} + \frac{y^2}{8} = 1$

(B) $\frac{x^2}{6} + \frac{y^2}{4} = 1$

(C) $\frac{x^2}{36} + \frac{y^2}{16} = 1$

(D) $\frac{x^2}{\sqrt{6}} + \frac{y^2}{2} = 1$

_____ 11. What is the eccentricity of the ellipse whose standard equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) 15   (D) $\frac{5}{3}$ (E) $\frac{5}{2}$

$a = 5, b = 4$
$a^2 - b^2 = c^2$
$25 - 16 = c^2$
$c^2 = 9$
$c = 3$

$e = \frac{c}{a}$

$e = \frac{3}{5}$ or 0.6

B 12. Convert the given general equation of the ellipse to the standard equation of the ellipse:

$$36x^2 + 16y^2 + 72x + 96y - 396 = 0$$

$36(x^2 + 2x + 1) + 16(y^2 + 6y + 9) = 396 + 36 + 144$
$36(x+1)^2 + 16(y+3)^2 = 576$
$\frac{(x+1)^2}{16} + \frac{(y+3)}{36} = 1$

(A) $\frac{(x-1)^2}{16} + \frac{(y-3)^2}{36} = 1$

(B) $\frac{(x+1)^2}{16} + \frac{(y+3)^2}{36} = 1$

(C) $\frac{(x+3)^2}{16} + \frac{(y+1)^2}{36} = 1$

(D) $\frac{(x+1)^2}{36} + \frac{(y+3)^2}{16} = 1$

C

13. (Calculator permitted) The reflective property of an ellipse is used in lithotripsy. Lithotripsy is a technique for treating kidney stones without surgery. Instead, high-energy shock waves are emitted from one focus of an elliptical shell and reflected painlessly to a patient's kidney stone located at the other focus. The vibration from the shock waves shatters the stone into pieces small enough to pass through the patient's urine naturally.
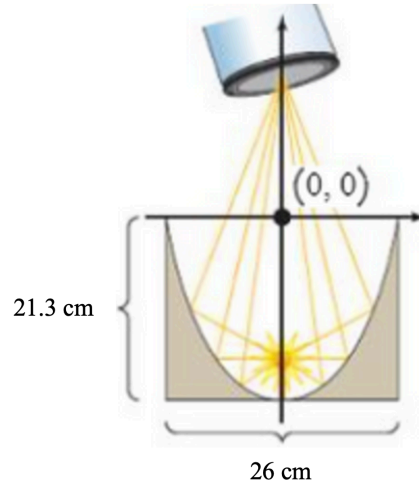
A vertical cross section of a lithotripter is in the shape of a semiellipse with the dimensions shown above. Approximate the distance from the center along the major axis where the patient's kidney stone should be located so the shock waves will target the stone. Round to two decimal places.

(A) 14.91 cm below the center   (B) 40.57 cm below the center
(C) 16.87 cm below the center   (D) 24.95 cm below the center

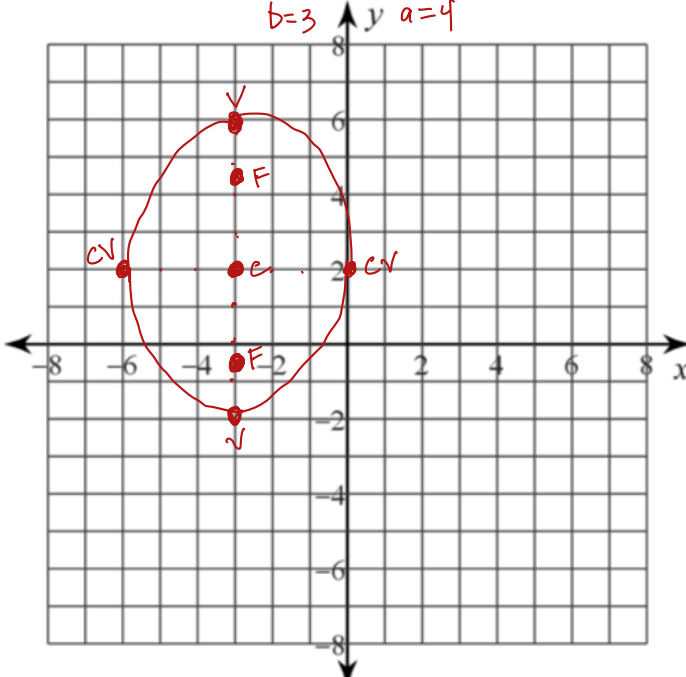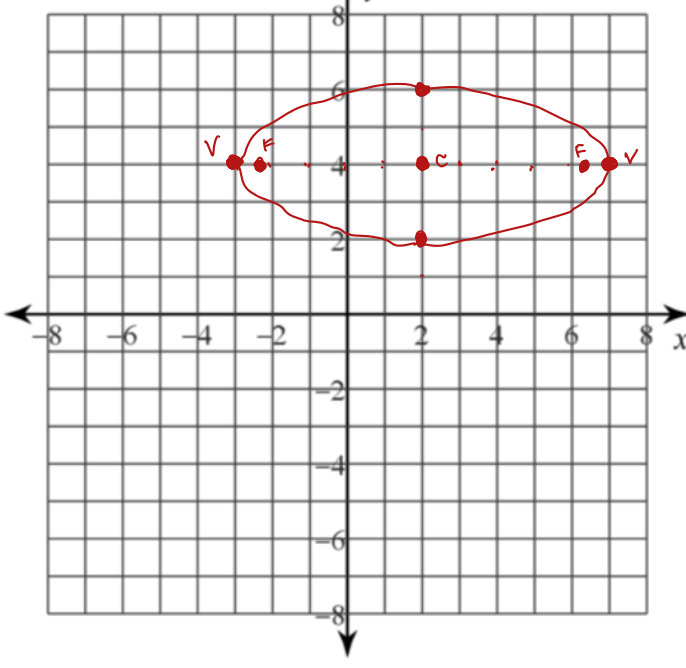Find c: $a = 21.3$ $b = \frac{26}{2}$
$= 13$

$a^2 - b^2 = c^2$

$(21.3)^2 - (13)^2 = c^2$

$c^2 = 284.69$

$c = 16.872 \text{ cm}$

Short Answer

14. Determine whether the ellipse is vertical or horizontal. Find the coordinates of the center, vertices, covertices, and foci of the ellipse with the given equation. Then sketch its graph. Find the eccentricity, domain, and range.

(a) $\dfrac{(x+3)^2}{9} + \dfrac{(y-2)^2}{16} = 1$

$b=3 \quad a=4$

Vertical or Horizontal: __Vertical__

Center: __$(-3, 2)$__

Vertices: __$(-3,-2) \,\&\, (-3, 6)$__

Covertices: __$(-6,2) \,\&\, (0,2)$__

Foci: __$(-3, 2-\sqrt{7}) \,\&\, (-3, 2+\sqrt{7})$__

Eccentricity: __$\dfrac{\sqrt{7}}{4} \approx 0.661$__

Domain: __$x \in [-6, 0]$__

Range: __$y \in [-2, 6]$__

$16 - 9 = c^2 \qquad e = \dfrac{c}{a} = \dfrac{\sqrt{7}}{4}$
$c = \sqrt{7} \approx 2\ldots$

(b) $\dfrac{(x-2)^2}{25} + \dfrac{(y-4)^2}{4} = 1$

$a=5 \quad b=2$

Vertical or Horizontal: __Horz__

Center: __$(2, 4)$__

Vertices: __$(-3,4) \,\&\, (7,4)$__

Covertices: __$(2,2) \,\&\, (2,6)$__

Foci: __$(2-\sqrt{7}, 4, \, 2+\sqrt{7}, 4)$__

Eccentricity: __$\dfrac{\sqrt{21}}{5} \approx 0.916$__

Domain: __$x \in [-3, 7]$__

Range: __$y \in [2, 6]$__

$25 - 4 = c^2 \qquad e = \dfrac{\sqrt{21}}{5}$
$c = \sqrt{21} \approx 4\ldots$

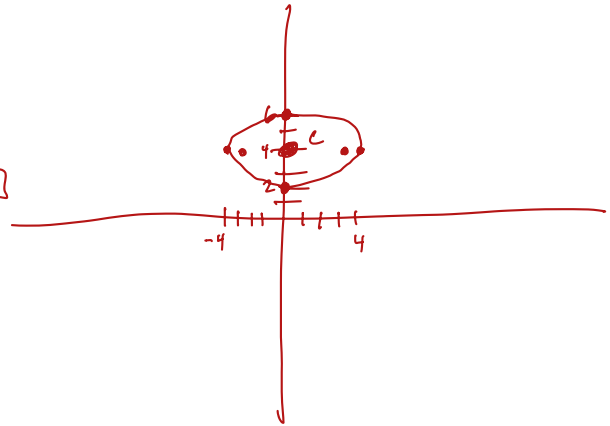15. Determine the Center, Vertices, Covertices, Foci, eccentricity, Domain, and Range of each ellipse. Then graph.

(a) $x^2 + 4y^2 - 32y + 48 = 0$

$x^2 + 4(y^2 - 8y + \underline{16}) = -48 + 64$

$x^2 + 4(y-4)^2 = 16$

$\dfrac{x^2}{16} + \dfrac{(y-4)^2}{4} = 1$

Center @ $(0,4)$ | $V(-4,4)$ & $(4,4)$ | $D: x \in [-4,4]$

$a = 4 \quad b = 2$ | $CV(0,2)$ & $(0,6)$ |

$16 - 4 = c^2$ | $F(-2\sqrt{3}, 4, 2\sqrt{3}, 4)$ | $R: y \in [2,6]$

$c = \sqrt{12}$

$c = 2\sqrt{3}$ | $e = \dfrac{2\sqrt{3}}{4}$

$= \dfrac{\sqrt{3}}{2}$

(b) $y^2 + 8x - 48 = 8y - 2x^2$

$2x^2 + 8x + y^2 - 8y = 48$

$2(x^2 + 4x + \underline{4}) + y^2 - 8y + \underline{16} = 48 + \underline{8} + \underline{16}$

$2(x+2)^2 + (y-4)^2 = 72$

$\dfrac{(x+2)^2}{36} + \dfrac{(y-4)^2}{72} = 1$ | $F(-2,-2)$ & $(-2,10)$

$b = 6 \qquad a = \sqrt{72}$ | $e = \dfrac{6}{6\sqrt{2}}$

$a = 6\sqrt{2}$ | $e = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$c = \sqrt{72 - 36}$ | $D: x \in [-8,4]$

$c = 6$ | $R: y \in [4 - 6\sqrt{2}, 4 + 6\sqrt{2}]$

Center @ $(-2,4)$

$V(-2, 4 - 6\sqrt{2})$ & $(-2, 4 + 6\sqrt{2})$

$CV(-8,4)$ & $(4,4)$
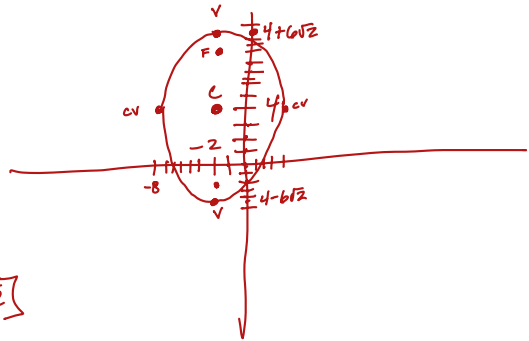
(c) $x^2 + 4y^2 + 13 = 8y + 10x$

$x^2 - 10x + \underline{25} + 4(y^2 - 2y + \underline{1}) = -13 + \underline{25} + \underline{4}$

$(x-5)^2 + 4(y-1)^2 = 28$

$\dfrac{(x-5)^2}{28} + \dfrac{(y-1)^2}{7} = 1$

$C(5,1) \qquad a = \sqrt{28} \quad b = \sqrt{7}$

$= 2\sqrt{7}$

≈ 5.29

$28 - 7 = c^2$

$c = \sqrt{21}$

$V(5 - 2\sqrt{7}, 1)(5 + 2\sqrt{7}, 1)$

$CV(5, 1 - \sqrt{7})(5, 1 + \sqrt{7})$ | $D: x \in [5 - 2\sqrt{7}, 5 + 2\sqrt{7}]$

$F(5 - \sqrt{21}, 1), (5 + \sqrt{21}, 1)$ | $R: y \in [1 - \sqrt{7}, 1 + \sqrt{7}]$

$e = \dfrac{\sqrt{21}}{2\sqrt{7}}$

$= \dfrac{\sqrt{3}}{2}$
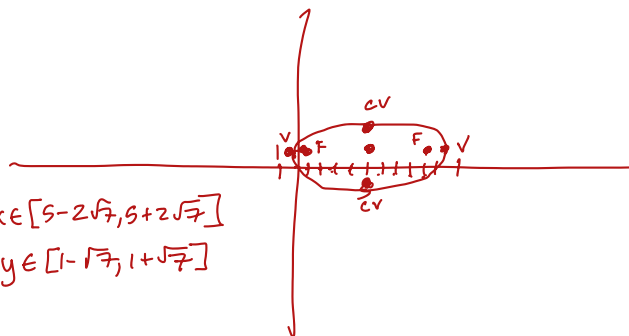
"straight side" in Latin

16. The **latus rectum** of an ellipse is the chord through either focus perpendicular to the major axis. In terms of a and b, what is its length for the ellipse with the standard equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1?$$

Latus Rectum = $2p$

$A (c, p)$

$F_2 (c, 0)$

$(x, -p)$

$* \ a^2 - b^2 = c^2$

75

$C(c, p)$ $\dfrac{c^2}{a^2} + \dfrac{p^2}{b^2} = 1$

$\dfrac{c^2}{a^2} + \dfrac{p^2}{b^2} = 1$

$\dfrac{p^2}{b^2} = 1 - \dfrac{c^2}{a^2}$

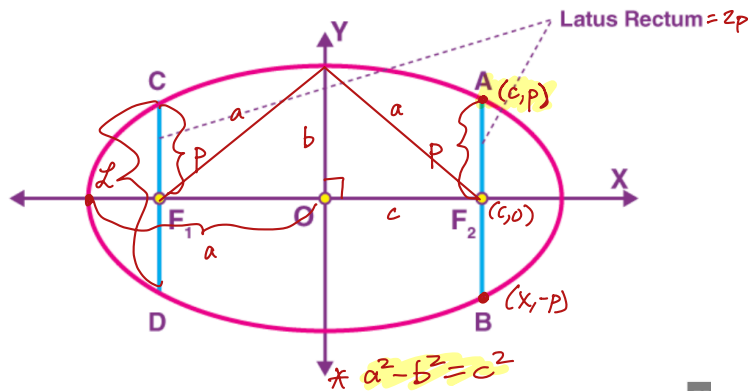$\dfrac{p^2}{b^2} = \dfrac{a^2 - c^2}{a^2}$

$* \ \dfrac{p^2}{b^2} = \dfrac{a^2 - (a^2 - b^2)}{a^2}$

$\dfrac{p^2}{b^2} = \dfrac{b^2}{a^2}$

$p^2 = \dfrac{b^4}{a^2}$

$p = \dfrac{b^2}{a}$

So Latus Rectum, $\mathscr{L} = 2p$

is $\mathscr{L} = \dfrac{2b^2}{a}$

17. *Astronomy*   The mean distance fromt the sun to Jupiter is 484 million miles (the mean distance is half the distance from its perihelion to its aphelion). If the eccentricity of Jupiter's orbit is 0.05, determine the minimum distance that Jupiter orbits from the sun at its perihelion.
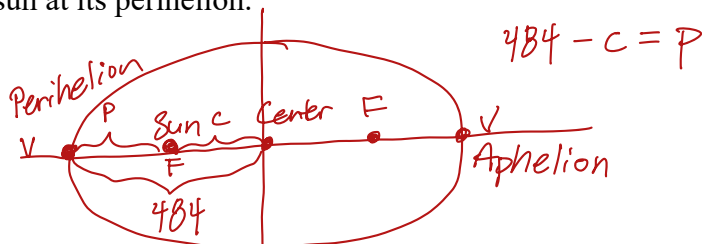
So $a = 484$ million

$e = \dfrac{c}{a} = 0.05$, find p

$\dfrac{c}{484} = 0.05$

$c = 24.2$ million

$p = 484 - 24.2$

$p = 459.8$ million miles

$484 - c = p$

Perihelion

Sun c Center F

Aphelion

484

So, at its closest, Jupiter is still 459.8 million miles from the sun

18. Write the standard equation of the ellipse with the following characteristics.

(a) Center (2,3), foci (2,5) & (2,1), and minor axis of length 6

$6 = 2b$

$c = 2$

$b = 3$

$a^2 - b^2 = c^2$

$a^2 - 9 = 4$

$a = \sqrt{13}$

So, eq is $\dfrac{(x-2)^2}{9} + \dfrac{(y-3)^2}{13} = 1$

(b) Foci (6,2) & (−2,2) and major axis of length of 10

$2c = 8$

$c = 4$

$2a = 10$

$a = 5$

Center @ (2,2)

$a^2 - b^2 = c^2$

$25 - b^2 = 16$

$b^2 = 9$

$b = 3$

So, eq is $\dfrac{(x-2)^2}{25} + \dfrac{(y-2)^2}{9} = 1$

19. A tunnel through a mountain for a four-lane highway is to have a semi-elliptical opening. The total width of the highway (not the opening) is to be 48 ft, and the height at the edge of the road must be sufficient for a truck 11 ft high to clear. If the highest point of the opening is to be 15 ft, approxiimately how wide must the opening be?
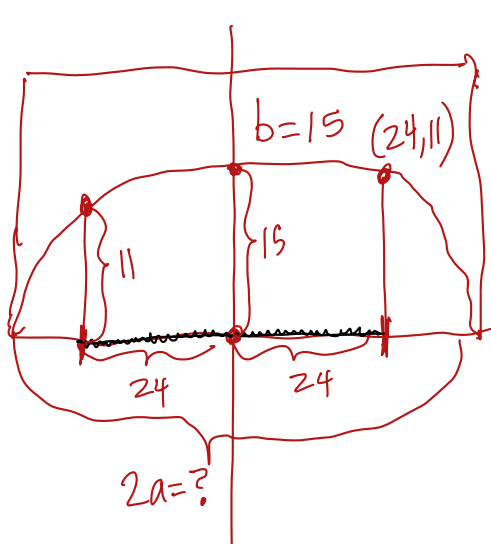
Center @ $(0,0)$, horiz

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{225} = 1$$

@ $(24, 11)$: $\frac{24^2}{a^2} + \frac{11^2}{225} = 1$

$$\frac{24^2}{a^2} = \frac{104}{225}$$

$$\frac{a^2}{24^2} = \frac{225}{104}$$

$$a = \sqrt{\frac{24^2 \cdot 225}{104}}$$

$a = 35.3009...$ ft

So, the opening must be at least $2a = 70.6018$ ft wide

(figure: semi-elliptical tunnel opening with $b=15$, point $(24,11)$, heights 11 and 15, widths 24 and 24, $2a=?$)

20. Find the intersection points of the pair of ellipses. Sketch the graphs of each ellipse on the same coordinate axes and label the points of intersection.

$$\begin{cases} \dfrac{x^2}{16} + \dfrac{y^2}{9} = 1 \text{ Blue} \\[2mm] \dfrac{x^2}{9} + \dfrac{y^2}{16} = 1 \text{ Green} \end{cases}$$

$x^2 = 16\left(1 - \frac{y^2}{9}\right)$ ①

$x^2 = 9\left(1 - \frac{y^2}{16}\right)$ ②

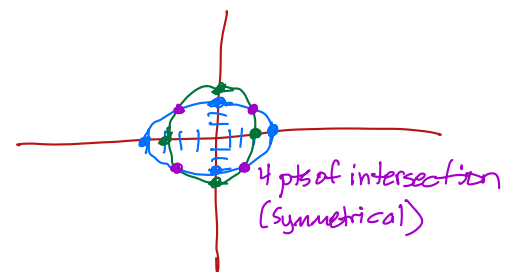① = ②: $16\left(1 - \frac{y^2}{9}\right) = 9\left(1 - \frac{y^2}{16}\right)$

$16 - \frac{16}{9}y^2 = 9 - \frac{9}{16}y^2$

$\left(-\frac{16}{9} + \frac{9}{16}\right)y^2 = 9 - 16$

$-\frac{175}{144}y^2 = -7$

$y^2 = \frac{144}{25}$

$y = \pm\frac{12}{5} \rightarrow$ ①

$x^2 = 16\left(1 - \frac{\left(\frac{12}{5}\right)^2}{9}\right)$

$x^2 = \frac{144}{25}$

$x = \pm\frac{12}{5}$

So, the intersection pts. are

$\left(\pm\frac{12}{5}, \pm\frac{12}{5}\right)$

4 pts of intersection (symmetrical)