Name KEY Date______________ Period______

Worksheet 9.3—Conic Sections: Hyperbolas

Show all work. No calculator is permitted, unless explicitly stated.

Multiple Choice

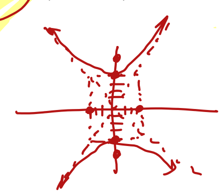
D 1. A hyperbola has 2 vertices and 2 foci.

(A) two, one (B) one, one (C) one, two (D) two, two

C 2. Find the coordinates of the foci of the hyperbola $-\frac{x^2}{9}+\frac{y^2}{16}=1$

$b=3$ $a=4$

(A) $(\pm 5,0)$ (B) $(\pm 4,0)$ (C) $(0,\pm 5)$ (D) $(0,\pm 4)$



$a^2+b^2=c^2$
$16+9=c^2$
$c=5$

D 3. What is the eccentricity for the hyperbola whose equation is $\frac{x^2}{9}-\frac{y^2}{16}=1$

$a=3$ $b=4$

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) 15 (D) $\frac{5}{3}$

$a^2+b^2=c^2$
$9+16=c^2$
$c=5$

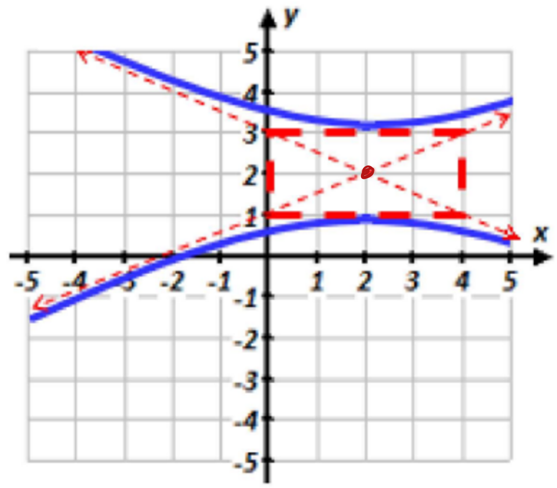
$e=\frac{c}{a}$
$=\frac{5}{3}$

C 4. What is the length of the conjugate axis for the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$? $b=4$

(A) 5 units (B) 4 units (C) 8 units (D) 10 units

conjugate axis length $=2b$
$=2(4)$
$=8$

A 5. Which equation best represents the graph of the hyperbola at right?



(A) $-\frac{(x-2)^2}{4}+\frac{(y-2)^2}{1}=1$

(B) $\frac{(x+2)^2}{2}-\frac{(y+2)^2}{1}=1$

(C) $\frac{(x-2)^2}{4}-\frac{(y-2)^2}{2}=1$

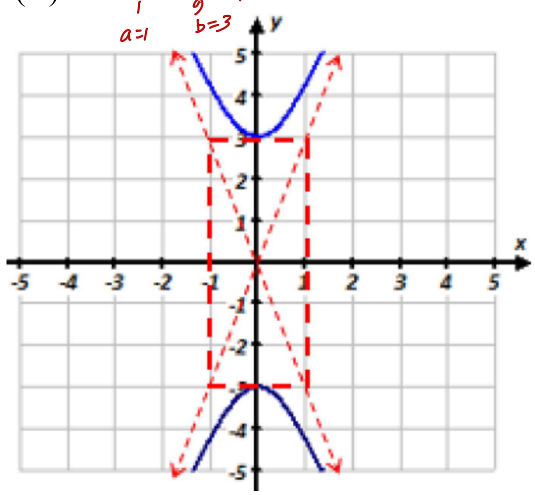
(D) $-\frac{(x+2)^2}{4}+\frac{(y+2)^2}{2}=1$

D 6. Graph $9y^2-x^2=9$

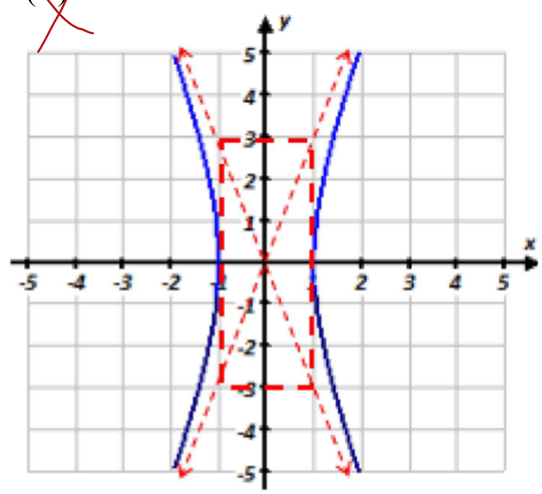
$\frac{y^2}{1}-\frac{x^2}{9}=1$

$a=1$ $b=3$

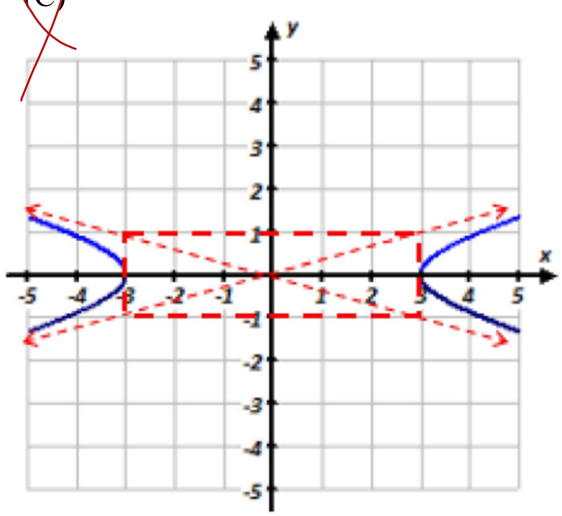
(A)



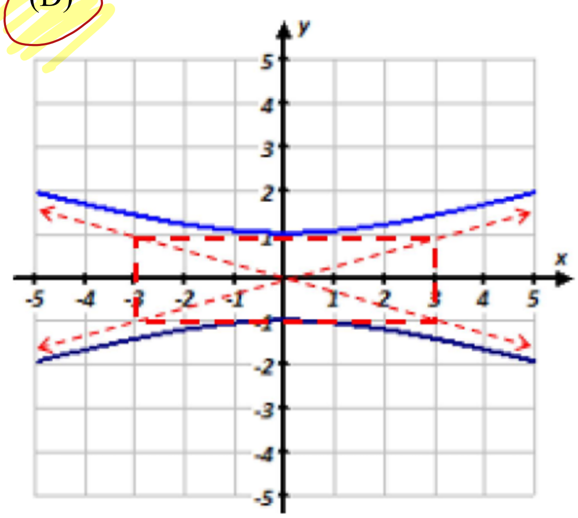
(B)



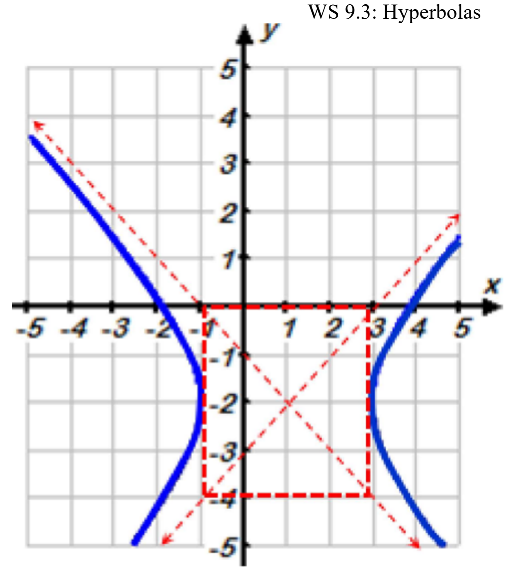
(C)



(D)



A 7. What are the equation of the asymptotes for the graph of the hyperbola at right?



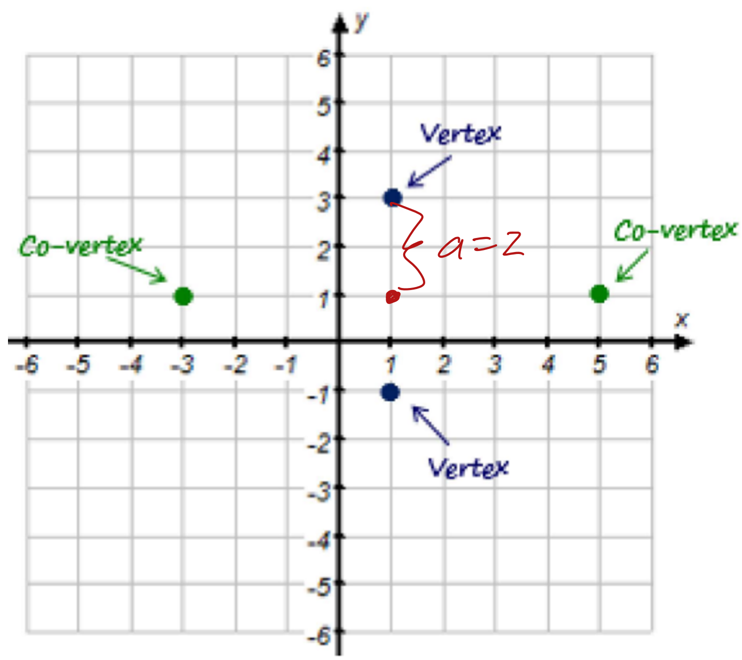
(A) $y = x - 3$ & $y = -x - 1$

(B) $y = x - 1$ & $y = -x - 3$

(C) $y = x + 3$ & $y = -x - 1$

(D) $y = -x + 3$ & $y = x - 1$

B 8. What is the standard equation of the hyperbola with the following characteristics?



(A) $\frac{(x-1)^2}{16} - \frac{(y-1)^2}{4} = 1$

(B) $-\frac{(x-1)^2}{16} + \frac{(y-1)^2}{4} = 1$

(C) $\cancel{\frac{(x+1)^2}{4}} - \frac{(y+1)^2}{16} = 1$

(D) $-\cancel{\frac{(x+1)^2}{4}} + \frac{(y+1)^2}{16} = 1$

D 9. Most basketball nets are approximately in the shape of a hyperboloid to catch and slow the basketball going through the hoop. Which equation best describes the hyperbola that outlines the basketball in the grid at right? (grid units are in inches)

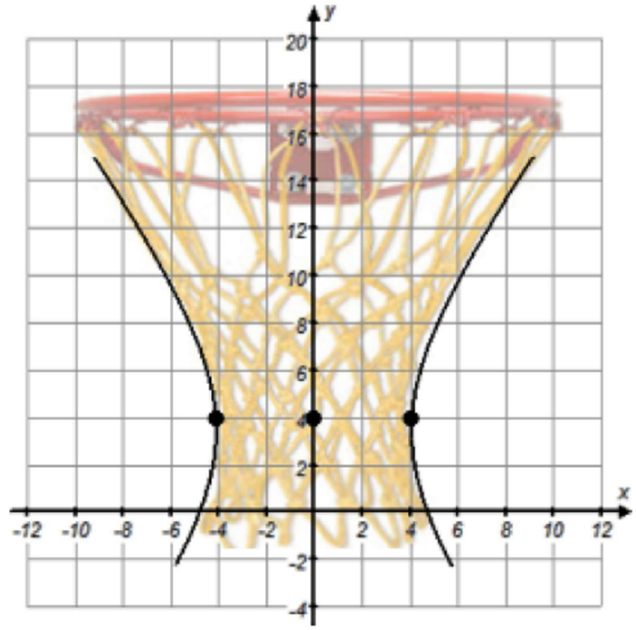
(A) $-\frac{x^2}{16}+\frac{(y+4)^2}{64}=1$

(B) $-\frac{x^2}{64}+\frac{(y+4)^2}{16}=1$

(C) $\frac{x^2}{16}-\frac{(y+4)^2}{64}=1$

(D) $\frac{x^2}{16}-\frac{(y-4)^2}{64}=1$

Center @ $(0,4)$



C 10. Find the center and vertices of the hyperbola $11x^2-25y^2+22x+250y-889=0$.

$11(x^2+2x+\underline{1})-25(y^2+10y+\underline{25})=889+\underline{11}-\underline{625}$

(A) Center: $(\cancel{1},-5)$, Vertices: $(1,-10)$ & $(1,0)$

(B) Center: $(-1,5)$, Vertices: $(-1,0)$ & $(-1,10)$

(C) Center: $(-1,5)$, Vertices: $(-6,5)$ & $(4,5)$

(D) Center: $(\cancel{1},-5)$, Vertices: $(-4,5)$ & $(6,-5)$

$11(x+1)^2-25(y+5)^2=$

$\frac{(x+1)^2}{25}-\frac{(y+5)^2}{11}=1$

$a=5$

$c(-1,-5)$

$V(-1+5,-5)(-1-5,-5)$

$V(4,-5)(-6,-5)$

A 11. Find the vertices and asymptotes of the hyperbola $9y^2-16x^2=144$.

$\frac{y^2}{16}-\frac{x^2}{9}=1$ $m=\frac{4}{3}$

$a=4$ $b=3$

(A) Vertices: $(0,\pm 4)$, Asymptotes: $y=\pm\frac{4}{3}x$

(B) Vertices: $(0,\pm 4)$, Asymptotes: $y=\pm\frac{3}{4}x$

(C) Vertices: $(\pm\cancel{4},0)$, Asymptotes: $y=\pm\frac{4}{3}x$

(D) Vertices: $(\pm\cancel{4},0)$, Asymptotes: $y=\pm\frac{3}{4}x$

C 12. Find the standard equation of the hyperbola with the given characteristics:

Vertices: $(0,\pm 6)$ Foci: $(0,\pm 7)$

$a=6$ $c=7$

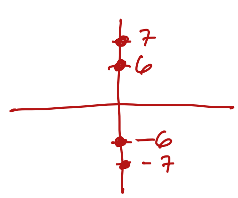
$a^2+b^2=c^2$

$36+b^2=49$

$b^2=13$

(A) $-\frac{x^2}{49}+\frac{y^2}{36}=1$

(B) $\frac{x^2}{36}-\frac{y^2}{13}=1$



(C) $-\frac{x^2}{13}+\frac{y^2}{36}=1$

(D) $\frac{x^2}{36}-\frac{y^2}{13}=49$

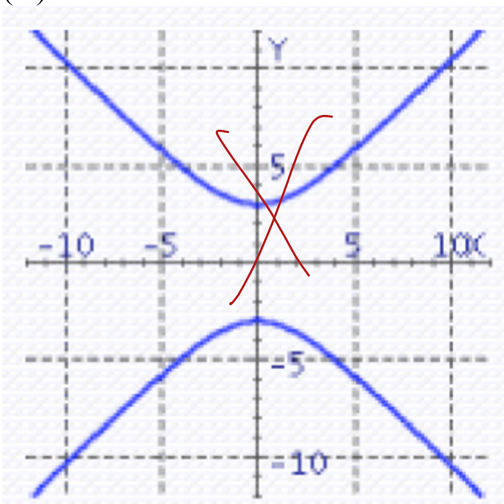
B 13. Graph the Square Hyperbola (transverse and conjugate axis are same length).

$$9x^2 - 9y^2 = 81$$

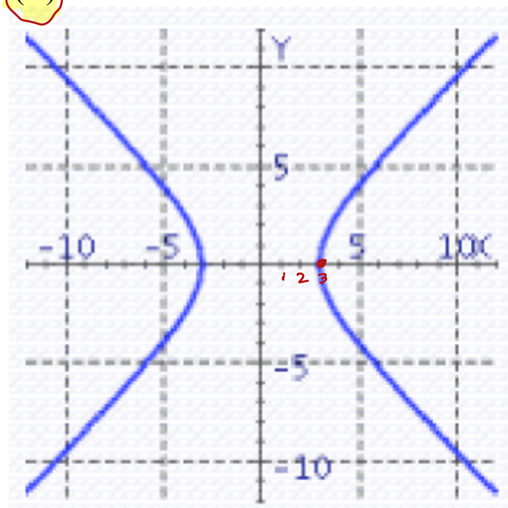
$\frac{x^2}{9} - \frac{y^2}{9} = 1$

$a=3$ $b=3$

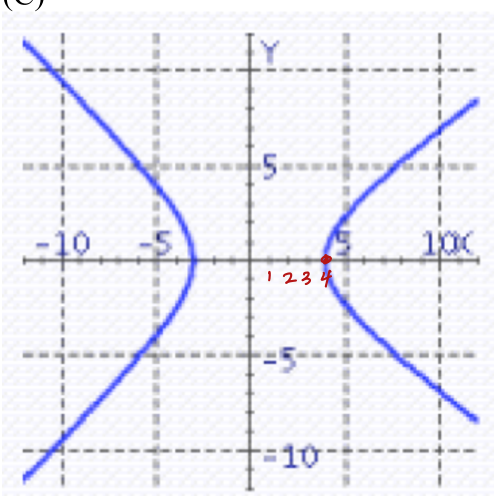
(A)



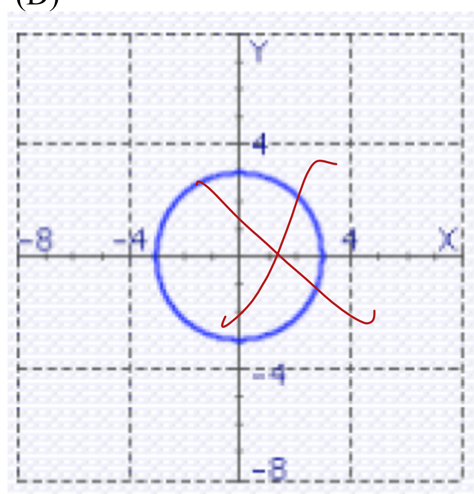
(B)



(C)



(D)



Short Answer

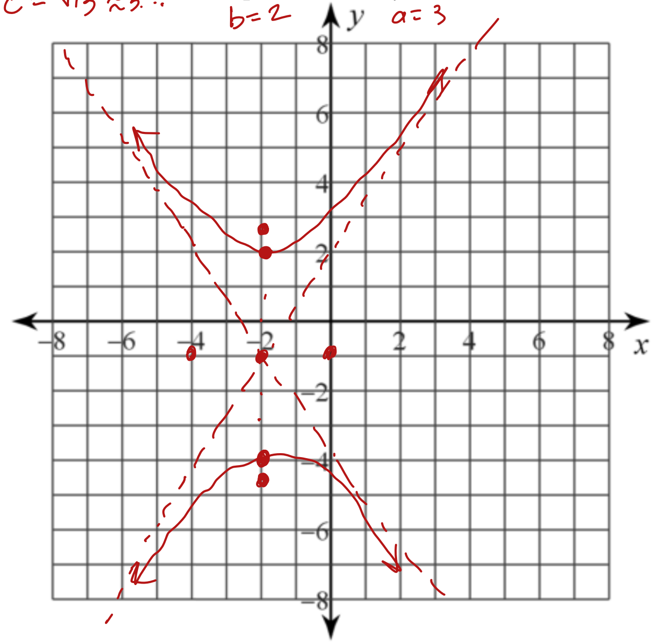
14. Determine if the hyperbola, whose equation is given below, is vertical or horizontal. Find the coordinates of the center, vertices, covertices, foci, eccentricity, the domain, range, and the equation of the slant asymptotes. Then sketch its graph.

$c^2 = 9+4$

$c = \sqrt{13} \approx 3..$

$$-\frac{(x+2)^2}{4} + \frac{(y+1)^2}{9} = 1$$

$b=2$ $a=3$



Vert or Horz: Vert

Center: $(-2,-1)$

Vertices: $(-2,2),(-2,-4)$

Covertices: $(0,-1)$ & $(-4,-1)$

Foci: $(-2,-1+\sqrt{13}),(-2,-1-\sqrt{13})$

Eccentricity: $e = \frac{\sqrt{13}}{3} \approx 1.201$

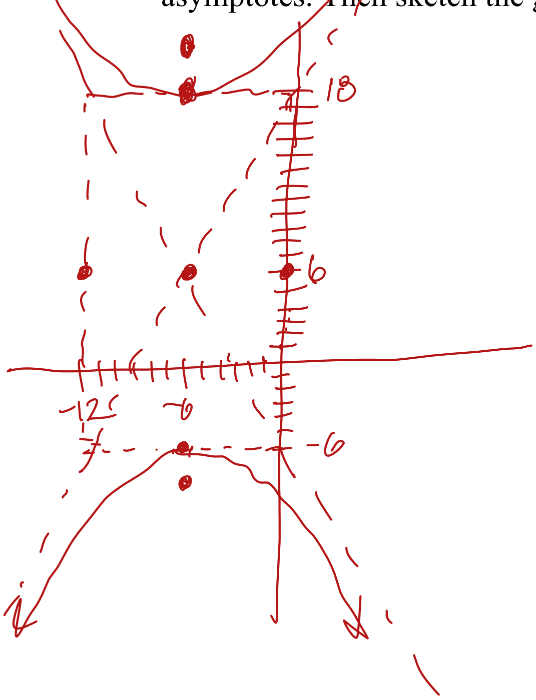
Domain: $\mathbb{R}$

Range: $y \in (-\infty,-4] \cup [2,\infty)$

SAs: $y = -1 \pm \frac{3}{2}(x+2)$

15. Write the equation of the given Hyperbola in standard form. Find the coordinates of the center, vertices, covertices, and foci, then find the eccentricity, the domain, range, and the equation of the slant asymptotes. Then sketch the graph.

$$-4x^2 + y^2 - 48x - 12y - 252 = 0$$



$-4(x^2+12x+\underline{36}) + y^2 - 12y + \underline{36} = 252 \underline{-144} + \underline{36}$

$-4(x+6)^2 + (y-6)^2 = 144$

$-\frac{(x+6)^2}{36} + \frac{(y-6)^2}{144} = 1$ (Vert)

$b=6$ $a=12$

$c^2 = 144+36$

$c = \sqrt{180}$

$c = 6\sqrt{5}$

SA @ $y = 6 \pm 2(x+6)$

$e = \frac{6\sqrt{5}}{12} = \frac{\sqrt{5}}{2} \approx 1.118$

Center @ $(-6,6)$

Vertse $(-6,-6),(-6,18)$

Covartse $(-12,6)(0,6)$

Foci @ $(-6,6-6\sqrt{5}),(-6,6+6\sqrt{5})$

D: $\mathbb{R}$, R: $y \in (-\infty,-6] \cup [18,\infty)$

16. Write the equation of the given Hyperbola in standard form. Find the coordinates of the center, vertices, covertices, and foci, then find the eccentricity, the domain, range, and the equation of the slant asymptotes. Then sketch the graph.

$$9x^2 - 16y^2 - 36x - 64y + 116 = 0$$

$9(x^2-4x+4)-16(y^2+4y+4)=-116+36-64$

$9(x-2)^2-16(y+2)^2=-144$

$-\frac{(x-2)^2}{16}+\frac{(y+2)^2}{9}=1$ (Vert)

$b=4$ $a=3$ $c^2=16+9$ $c=5$

Center @ $(2,-2)$

Vert @ $(2,-5),(2,1)$

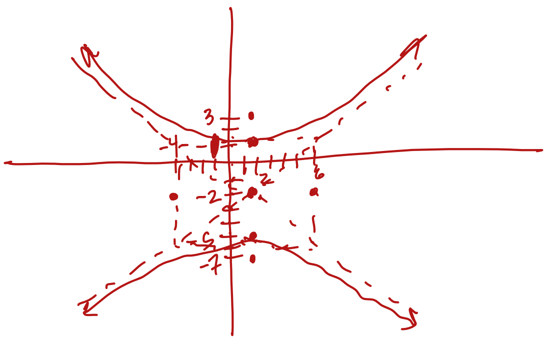
Coverts @ $(-4,-2),(6,-2)$

Foci @ $(2,-7),(2,3)$

$e=\frac{5}{3}\approx 1.666$

SAs @ $y=-2\pm\frac{3}{4}(x-2)$

D: $\mathbb{R}$, R: $y\in(-\infty,-5]\cup[1,\infty)$

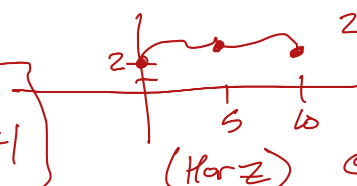


17. Determine the standard equation of the Hyperbola whose foci are at (0,2) & (10,2) with a transverse axis of length 8.

$2a=8$
$a=4$

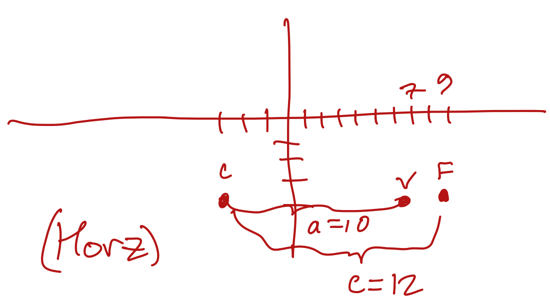
$a^2+b^2=c^2$
$16+b^2=25$
$b^2=9$
$b=3$

$$\boxed{\frac{(x-5)^2}{16}-\frac{(y-2)^2}{9}=1}$$



$2c=10$
$c=5$
(Horz) center @ $(5,2)$

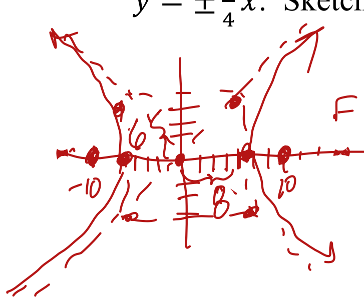
18. Determine the standard equation of the Hyperbola whose center is at $(-3,-4)$ with one vertex at $(7,-4)$ and one focus at $(9,-4)$.



$a^2+b^2=c^2$
$100+b^2=144$
$b^2=44$
$b=2\sqrt{11}$

$$\boxed{\frac{(x+3)^2}{100}-\frac{(y+4)^2}{44}=1}$$

19. Find the standard equation of the hyperbola if a a focus point is at (10,0) and the asymptotes are $y=\pm\frac{3}{4}x$. Sketch the graph of this hyperbola with its asymptotes and foci.



$a=4$
$b=3$
$c=10$

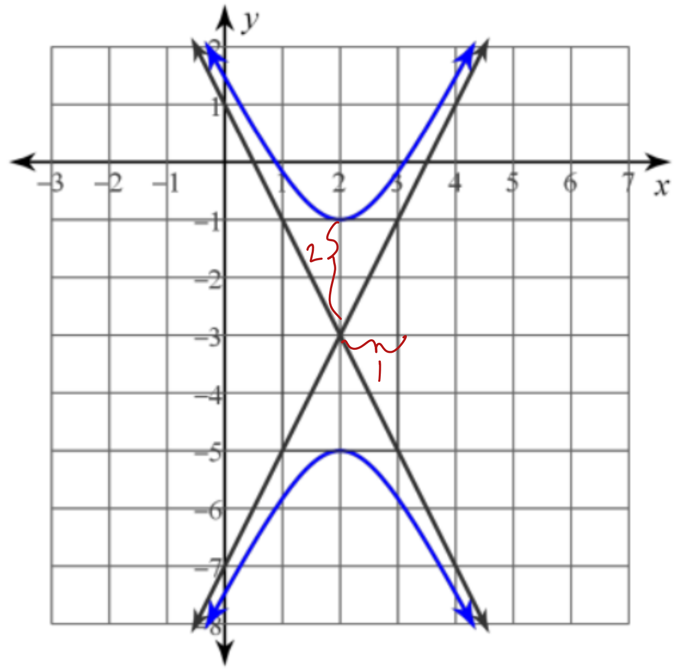
(Horz)
Center @ $(0,0)$

$$\boxed{\frac{x^2}{64}-\frac{y^2}{36}=1}$$

$m=\frac{3}{4}=\frac{6}{8}\begin{matrix}=b\\=a\end{matrix}$

36 64

20. Write the standard equation of the Hyperbola whose graph is given below.



Center @ $(2,-3)$

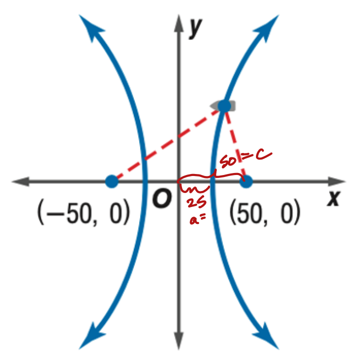
$a=2,\ b=1$

$4+1=c^2$

$c=\sqrt{5}$

$$\boxed{-\frac{(x-2)^2}{1}+\frac{(y+3)^2}{4}=1}$$

21. *Navigation* The LORAN (LOng RAnge Navigation) navigational system is based on hyperbolas. Two stations send out signals at the same time. A ship notes the difference in the times at which it receives the signals. The ship is on a hyperbola with the stations at the foci. Suppose a ship determines that the difference of its distances from two stations is 50 nautical miles. The stations are 100 nautical miles apart. Write an equation for the hyperbola on which the ship lies if the stations are $(-50,0)$ and $(50,0)$. (if a third station is used, in conjunction with one of the first two, the intersection of the two hyperbolas pinpoints the ship by triangulation.) *Hint: remember from the notes what constant the difference of the distances equals and how it relates to the vertices).*



$2a = \text{diff of distances} = 50$, $c=50$

$a=25$

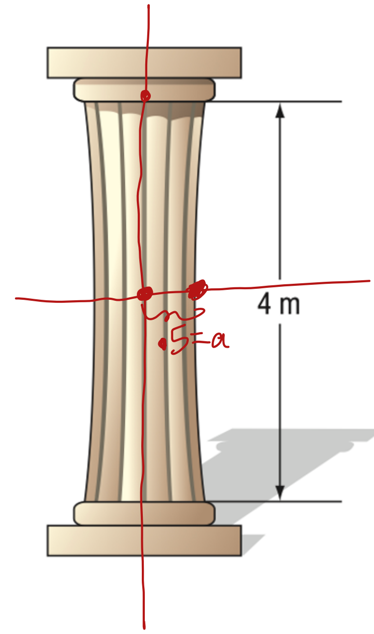
$$\boxed{\frac{x^2}{625}-\frac{y^2}{1875}=1}$$

$a^2+b^2=c^2$

$25^2+b^2=50^2$

$b^2=1875$

22. *Structural Design* (Calculator permitted) An architect's design for a building includes some large pillars with cross sections in the shape of hyperbolas. The curves can be modeled by the equation $\frac{x^2}{0.25} - \frac{y^2}{9} = 1$, where the units are in meters. If the pillars are 4 meters tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to three decimals.



$a = 0.5$

$b = 3$

$c = \sqrt{9.25}$

At narrowest point, the pillar is

$2a = 0.5(2) = 1$ meter wide

when $y=4$: $\frac{x^2}{0.25} - \frac{16}{9} = 1$

$$\frac{x^2}{0.25} = \frac{25}{9}$$

$$x^2 = \frac{25}{36}$$

$x = \frac{5}{6}$ meter

So, the widest the pillar is at the top is $2x = \frac{5}{3} \approx 1.666$ meters