## 

## Pritenometis Ratios

Today we look at three special ratios of right triangles. The word Trigonometry is derived from two Greek words meaning "angle measure." When we look at the ratio of any two sides of a given right triangle, we know its relative side, which depends entirely on the size of the reference angle.

Remember that we can label the vertices of a triangle as capital letters, and the line segments opposite the angles as the corresponding lower-case letters.


We can say that the size of the triangle is dependent on the independent size of the reference angle. Once we establish an angle's measure, the other angle and the side length's relative sizes are predetermined (remember similar triangles!).

In relation to the reference angle, how can we look at the ratio of two sides of the triangle at a time? Let's see:


$$
\begin{aligned}
& \frac{b}{c}=\frac{\text { opposite }}{\text { hypotenuse }} \\
& \frac{a}{c}=\frac{\text { adjacent }}{\text { hypotenuse }} \\
& \frac{b}{a}=\frac{\text { opposite }}{\text { adjacent }}
\end{aligned}
$$

There are three more ratios, which are merely the reciprocal of these, so for now, we will concentrate on these guys. We should probably give them some names!!!!


## Trig Ratio Names

| $\frac{\text { Ratio }}{}$ | Name | Abbreviation |
| :--- | :---: | :---: |
| $\frac{b}{c}=\frac{\text { opposite }}{\text { hypotenuse }}$ | sine of $\theta$ | $\sin \theta$ |
| $\frac{a}{c}=\frac{\text { adjacent }}{\text { hypotenuse }}$ | cosine of $\theta$ | $\cos \theta$ |
| $\frac{b}{a}=\frac{\text { opposite }}{\text { adjacent }}$ | tangent of $\theta$ | $\tan \theta$ |

*** $\theta$ is the reference angle of the RIGHT triangle. If the words "opposite" and "adjacent" are specific to a particular angle. Therefore, it is EXTREMELY important to carefully and deliberately choose a reference angle FIRST when setting up these ratios.

## Example:

Find the three trig ratios for the following right triangle for angle $A$, then angle $C$.

$\sin A=\frac{5}{5 \sqrt{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$
$\cos A=\frac{10}{5 \sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2 \sqrt{5}}{5}$
$\tan A=\frac{5}{10}=\frac{1}{2}$

$$
\sin C=\frac{10}{5 \sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2 \sqrt{5}}{5}
$$

$\cos C=\frac{5}{5 \sqrt{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$
$\tan C=\frac{10}{5}=2$
**Notice that the right angle cannot be the reference angle. The hypotenuse would also be the opposite side, and there would be two adjacent sides. Although three ratios exist for a right angle, it is a special case you will study in Precalculus!!

There is an easy way to remember the three ratios, a little mnemonic that former geometry students can still be heard saying to themselves years and years later . . .


SOH: Sine is Opposite over Hypotenuse CAH: Cosine is Adjacent over Hypotenuse TOA: Tangent is Opposite over Adjacent

If that doesn't work, you can use:
Some Out-Houses Can Actually Have Totally Odorless Aromas Some Old Horse Caught Another Horse Taking Oats Away !
Let's redefine these ratios now in terms of our new letters, $A, O, \& H$, and reference angle $\theta$.

$$
\sin \theta=\frac{O}{H} \quad \cos \theta=\frac{A}{H} \quad \tan \theta=\frac{O}{A}
$$

Now we can use the ratios as formulas to find missing measurements of a right triangle.

## Example:

Find the value of $x$.

Our reference angle is 32. We want to find the side opposite this, $O$, and we are given the hypotenuse, $H$. This means we use the trig ratio containing the $O$ and $H$, namely the Sine ratio.

$$
\sin \theta=\frac{O}{H} \rightarrow \sin 32^{\circ}=\frac{x}{25} \rightarrow x=25 \sin 32^{\circ} \approx 13.248
$$

Your calculator has a SIN, COS, and TAN button. Be sure that you are in degree MODE, and that you close the parenthesis after you type in the angle (the calculator automatically opens them)

You can also use the trig ratio formulas to find a missing angle.

## Example:

## Find the measure of angle $z$.



In reference to angle $z$, we are given the opposite, $O$, and the adjacent, $\boldsymbol{A}$, sides. This means we will use the tangent ratio:

$$
\tan \theta=\frac{O}{A} \rightarrow \tan z=\frac{8}{7} \rightarrow z=\tan ^{-1}\left(\frac{8}{7}\right)=48.814^{\circ}
$$

The inverse operation of taking the tangent of the angle is taking the INVERSE tangent of a ratio. On your calculator, you hit the " 2 nd" function button first, then the "TAN" button.

## 

## Application 1:

From the top of a 150-foot air traffic control tower, a controller spots a jet at an angle of $42^{\circ}$ above his line of site. If the controller's radar shows the jet at an altitude of 2600 feet, how far is the jet from the control tower?


In the triangle, the opposite side of the 42 angle is 2600-150=2450. We are trying to find the hypotenuse. We use the sine ratio:
$\sin 42^{\circ}=\frac{2450}{H} \rightarrow H=2450\left(\sin 42^{\circ}\right) \approx 1639$ feet

## Application 2:



A surveyor standing at the edge of a river made the measurements shown below. Find the distance across the river from $C$ to $B$ (without getting wet!!)


We want the opposite side, $O$, and we are given the adjacent, $A$. We use tangent. $\tan 64^{\circ}=\frac{O}{200} \rightarrow 0=200\left(\tan 64^{\circ}\right) \approx 410$ feet.

## Web resources:

http://www.glencoe.com
http://www.cmsairexpress.com/JET.gif
http://virtualskies.arc.nasa.gov/ATM/tutorial/tutorial4.html http://www.stvincent.ac.uk/Resources/Physics/Speed/speed/atc.html http://www.surveylee.com/

